

Composition and Structure of Protoneutron Stars

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Abstract

We investigate the structure of neutron stars shortly after they are born, when the entropy per baryon is of order 1 or 2 and neutrinos are trapped on dynamical timescales. We find that the structure depends more sensitively on the composition of the star than on its entropy, and that the number of trapped neutrinos play an important role in determining the composition. Since the structure is chiefly determined by the pressure of the strongly interacting constituents and the nature of the strong interactions is poorly understood at high density, we consider several models of dense matter, including matter with strangeness-rich hyperons, a kaon condensate and quark matter.

In all cases, the thermal effects for an entropy per baryon of order 2 or less are small when considering the maximum neutron star mass. Neutrino trapping, however, significantly changes the maximum mass due to the abundance of electrons. When matter is allowed to contain only nucleons and leptons, trapping decreases the maximum mass by an amount comparable to, but somewhat larger than, the increase due to finite entropy. When matter is allowed to contain strongly interacting negatively charged particles, in the form of strange baryons, a kaon condensate, or quarks, trapping instead results in an *increase* in the maximum mass, which adds to the effects of finite entropy. A net increase of order $0.2M_{\odot}$ occurs.

The presence of negatively-charged particles has two major implications for the neutrino signature of gravitational collapse supernovae. First, the value of the maximum mass will decrease during the early evolution of a neutron star as it loses trapped neutrinos, so that if a black hole forms, it either does so immediately after the bounce (accretion being completed in a second or two) or it is delayed for a neutrino diffusion timescale of ~ 10 s. The latter case is most likely if the maximum mass of the hot star with trapped neutrinos is near $1.5M_{\odot}$. In the absence of negatively-charged hadrons, black hole formation would be due to accretion and therefore is likely to occur only immediately after bounce. Second, the appearance of hadronic negative charges results in a general softening of the equation of state that may be observable in the neutrino luminosities and average energies. Further, these additional negative charges decrease the electron fraction and may be observed in the relative excess of electron neutrinos compared to other neutrinos.

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1 A neutron star is born

A protoneutron star is formed in the aftermath of the gravitational collapse of the core of a massive star. Its evolution proceeds through several distinct stages [1, 2], which may have various outcomes, as shown schematically in Fig. 1.

1. Immediately following core bounce and the passage of a shock through the outer protoneutron star's mantle, the star contains an unshocked, low entropy core of $0.5 - 0.7 M_{\odot}$ in which neutrinos are trapped [3, 4]. This is surrounded by a low density, high entropy mantle that is both accreting matter falling through the shock and rapidly losing energy due to beta decays and neutrino emission. The shock is momentarily stationary prior to an eventual explosion.
2. On a timescale of about 0.5 s, accretion becomes much less important as the supernova explodes and the shock lifts off the stellar envelope. Extensive neutrino losses and deleptonization of the mantle lead to the loss of lepton pressure and to collapse of the mantle on the same timescale. Neutrino diffusion times from the core are too long to significantly alter the core during this stage. If enough accretion occurs, and the initial core were large enough, the mass of the hot, lepton-rich matter could exceed the maximum mass which is stable, in which case the star would collapse to form a black hole. In this event, neutrino emission would effectively cease, since the event horizon is believed to form outside the neutrino photosphere [5].
3. This stage is dominated by neutrino diffusion causing deleptonization and heating of the core. Neutrino-nucleon absorption reactions set the diffusion timescale to about 10–15 s. The maximum entropy per baryon reached in the core is about 2 (in units of Boltzmann's constant). When the core deleptonizes, the threshold for the appearance of strangeness, in the form of hyperons, a Bose kaon condensate, or quarks, will be reduced [6, 7, 8, 9]. If one (or more) of these additional components is present, the equation of state will soften, leading to a decrease in the maximum mass. There is, therefore, the possibility that a black hole could form at this later time.
4. Following deleptonization, the star has a high entropy, so that thermally produced neutrino pairs of all flavors are abundant, and thermal diffusion and cooling of the hot neutron star takes place. Because the entropy is higher at the beginning of cooling than it is at the beginning of deleptonization, the neutrino mean free paths are smaller and the timescales longer. In approximately 50 s, as the average neutrino energy decreases, the star becomes essentially transparent to neutrinos and the core achieves a cold, catalyzed configuration. The loss of thermal energy leads to a small increase in the threshold density for the appearance of strange matter, so that, in the

absence of further accretion, it is unlikely that a black hole could form during this or later phases.

5. Following the onset of neutrino transparency, the core continues to cool by neutrino emission, but the star's crust cools less because of its lower neutrino emissivity. The crust acts as an insulating blanket which prevents the star from coming to complete thermal equilibrium and keeps the surface relatively warm ($T \approx 3 \times 10^6$ K) for up to 100 years. This timescale is primarily sensitive to the neutron star's radius and the thermal conductivity of the mantle [10].
6. Ultimately, the star achieves thermal equilibrium when the energy stored in the crust is depleted. The temperature to which the surface now cools is determined by the rate of neutrino emission in the star's core. If this rate is large, the surface temperature will become relatively small, and the photon luminosity may become virtually undetectable from the Earth. This will be the case if the direct Urca (beta decay) process can occur, which happens if the nuclear symmetry energy is large or if hyperons, a Bose condensate, or quarks are present. Somewhat higher surface temperatures occur if superfluidity in the core cuts off the direct Urca rate below the superfluid's critical temperature. A relatively high surface temperature will persist if the Urca process can only occur indirectly with the participation of a spectator nucleon – the so-called standard cooling scenario.

Neutrino observations from a galactic supernova will shed much light on the first four of the above stages. Observations of X-rays or γ -rays from very young neutron stars are crucial for the last stages. The duration of each of these stages is essentially determined by neutrino diffusion timescales, and thus depends both upon the microphysics and the macrophysical structure of neutron stars. Roughly, the diffusion timescale is proportional to $R^2(c\lambda)^{-1}$, where R is the star's radius and λ is the effective neutrino mean free path. Thus, important constraints upon the properties of dense matter can be achieved by looking at this relation as it applies to each stage. Generally, the structure (i.e., mass, radius, etc.) of both hot and cold, and both neutrino-rich and neutrino-poor, stars is fixed by the equation of state (EOS) and the composition. Both are also crucial to knowledge of the neutrino mean free paths.

The behavior of the maximum mass as a function of temperature and neutrino trapping is of practical importance [11]. If the maximum mass of a cold, neutrino-free neutron star is near the largest measured masses of neutron stars [12], namely 1.442 ± 0.001 M_\odot in the case of PSR 1913+16, the question arises as to whether or not a larger mass can be stabilized during the preceding evolution of a star from a hot, lepton-rich state. If it cannot, then any transition from a neutron star to a black hole in the stellar collapse process should occur extremely early, either immediately or during the first few tenths of a second, when the

proto-neutron star is rapidly accreting mass from unejected matter behind the shock. If a larger mass can be stabilized, however, the transition from a neutron star to a black hole, if it occurs at all, could happen later, on the neutrino diffusion or thermal timescale, namely ~ 10 seconds. Burrows [5] has demonstrated that the appearance of a black hole should be accompanied by a dramatic cessation of the neutrino signal (since the event horizon invariably forms outside the neutrinosphere).

A newly-formed neutron star should accrete its final baryon mass within a second or two of its birth, so that the neutrinos will not have had time to diffuse from the stellar core. Thus, *both* the maximum mass for a hot, neutrino-trapped star and a cold, catalyzed star must be greater than the largest measured neutron star mass. Moreover, due to the binding energy released because of neutrino emission and cooling, the maximum mass for a hot, neutrino-trapped star must be at least $0.1 - 0.2 M_\odot$ larger than this limit. This feature could provide a more severe constraint upon the equation of state than limits based solely upon cold, catalyzed matter. This is especially true if the neutrino-trapped maximum mass is less than the cold, catalyzed maximum mass for a given equation of state.

2 Scope of this work

The evolutionary scenario presented above is based on dynamical calculations [1] carried out with a schematic equation of state, since little work has been carried out on the structure of protoneutron stars shortly after their birth, although, of course, there have been many investigations of cold neutron stars. The purpose of this work is to investigate the composition and structure of these newly born stars. It would then be of interest to study the implications for the dynamical evolution of neutron stars, but this we defer to the future.

There are two new effects to be considered for a newborn star. Firstly, thermal effects which result in an approximately uniform entropy/baryon of 1–2 across the star [1]. Secondly, the fact that neutrinos are trapped in the star, which means that the neutrino chemical potential is non-zero and this alters the chemical equilibrium, which leads to compositional changes. Both effects may result in observable consequences in the neutrino signature from a supernova and may also play an important role in determining whether or not a given supernova ultimately produces a cold neutron star or a black hole.

Since the composition of a neutron star chiefly depends upon the nature of the strong interactions, which are not well understood in dense matter, we shall investigate many of the possible models. After a brief discussion of the equilibrium conditions in section 3, we begin section 4 by discussing non-relativistic potential models and their predictions for protoneutron stars. We then turn to relativistic models, which may be more appropriate, since the central densities involved are high. At first, we allow only nucleons to be present

in addition to the leptons. Neutrino trapping increases the electron chemical potential and, therefore, the lepton and (due to charge neutrality) the proton abundances. With more protons, the equation of state is softer and the maximum neutron star mass is lower. However, it has been recently realized that if additional negatively charged particles, such as kaons, hyperons or quarks, are present, this situation can be qualitatively changed [6, 7, 8, 9]. This is due to the change in the chemical potentials, which delays the appearance of these strongly interacting particles that lower the pressure. Their softening effect is therefore less in evidence, and a larger maximum mass is obtained for the young star. This means that the star could become unstable when the initial neutrino population has departed, as we mentioned in section 1. In the remainder of this section, we therefore investigate in some detail the effect of hyperons, kaon condensates, and quarks upon the structure of the protoneutron star.

The evolution of the protoneutron star is determined by the time scales involved. While accurate results require detailed numerical work that incorporates neutrino transport, it is nevertheless possible to use semi-analytical techniques to gain an understanding of the times involved and the general energetics. This is discussed in section 5.

In section 6, we discuss the implications of this work for delayed black hole formation and for neutrino signals from supernovae, in general, and SN1987A in particular.

3 Equilibrium conditions

For stars in which the strongly interacting particles are only baryons, the composition is determined by the requirements of charge neutrality and equilibrium under the weak processes

$$B_1 \rightarrow B_2 + \ell + \bar{\nu}_\ell; \quad B_2 + \ell \rightarrow B_1 + \nu_\ell, \quad (1)$$

where B_1 and B_2 are baryons, and ℓ is a lepton, either an electron or a muon. Under conditions when the neutrinos have left the system, these two requirements imply that the relations

$$\sum_i (n_{B_i}^{(+)} + n_{\ell_i}^{(+)}) = \sum_i (n_{B_i}^{(-)} + n_{\ell_i}^{(-)}) \quad (2)$$

$$\mu_i = b_i \mu_n - q_i \mu_\ell, \quad (3)$$

are satisfied. Above, n denotes the number density and the superscripts (\pm) on n signify positive or negative charge. The symbol μ_i refers to the chemical potential of baryon i , b_i is its baryon number and q_i is its charge. The chemical potential of the neutron is denoted by μ_n .

Under conditions when the neutrinos are trapped in the system, the beta equilibrium condition Eq. (3) is altered to

$$\mu_i = b_i \mu_n - q_i (\mu_\ell - \mu_{\nu_\ell}), \quad (4)$$

where μ_{ν_ℓ} is the chemical potential of the neutrino ν_ℓ . Because of trapping, the numbers of leptons per baryon of each flavor of neutrino, $\ell = e$ and μ ,

$$Y_{L\ell} = Y_\ell + Y_{\nu_\ell}, \quad (5)$$

are conserved on dynamical time scales. Gravitational collapse calculations of the white-dwarf core of massive stars indicate that at the onset of trapping, the electron lepton number $Y_{Le} = Y_e + Y_{\nu_e} \simeq 0.4$, the precise value depending on the efficiency of electron capture reactions during the initial collapse stage. Also, because no muons are present when neutrinos become trapped, the constraint $Y_{L\mu} = Y_\mu + Y_{\nu_\mu} = 0$ can be imposed. We fix $Y_{L\ell}$ at these values in our calculations for *neutrino trapped* matter.

For completeness, we give here the partition function for the leptons. Since their interactions give negligible contributions [13], it is sufficient to use the non-interacting form of the partition function:

$$\begin{aligned} \ln Z_L = & \frac{V}{T} \sum_i g_i \frac{\mu_i^4}{24\pi^2} \left[1 + 2 \left(\frac{\pi T}{\mu_i} \right)^2 + \frac{7}{15} \left(\frac{\pi T}{\mu_i} \right)^4 \right] \\ & + V g_\mu \int \frac{d^3 k}{(2\pi)^3} \left[\ln \left(1 + e^{-\beta(e_\mu - \mu_\mu)} \right) + \ln \left(1 + e^{-\beta(e_\mu + \mu_\mu)} \right) \right], \end{aligned} \quad (6)$$

where V is the volume and $\beta = T^{-1}$ is the inverse temperature. The first term gives the contribution of massless particles and antiparticles. Since $\mu_e \gg m_e$ in all regimes considered here, this term applies to both electrons and neutrinos. The degeneracies, g_i , are 2 and 1 for electrons and neutrinos, respectively. The second term gives the muon contributions, with degeneracy $g_\mu = 2$, $e_\mu = \sqrt{k^2 + m_\mu^2}$ and the muon chemical potential designated by μ_μ . The pressure, density and energy density of the leptons are obtained from Eq. (6) in the standard fashion. The total partition function, $Z_{\text{total}} = Z_H Z_L$, where Z_H is the partition function of the hadrons discussed below.

4 Stellar Composition and Structure

4.1 Models of hot and dense hadronic matter

The properties of neutron stars can be obtained from the well-known hydrostatic equilibrium equations of Tolman, Oppenheimer and Volkov [14], once the equation of state is

specified. At very low densities ($n < 0.001 \text{ fm}^{-3}$), we use the Baym-Pethick-Sutherland [15] EOS, while for densities $0.001 < n < 0.08 \text{ fm}^{-3}$, we employ the EOS of Negele and Vautherin [16]. At higher densities the EOS depends on the nature of the strong interactions. These are not yet known with certainty, although several intriguing possibilities are currently being investigated. In view of this, we will investigate the influence of thermal effects and neutrino-trapping on the structure of neutron stars by considering widely differing models of dense matter and trying to identify the common features shared by these models.

The models considered include (a) a generalization of a schematic potential model based on the work of Prakash, Ainsworth and Lattimer [17], which reproduces the results of more microscopic calculations [18] of dense matter, (b) a relativistic field theoretical model [19], based on the archetypal Walecka model [20], in which baryons interact via the exchange of σ -, ρ - and ω -mesons, (c) a model based on the chiral Lagrangian of Kaplan and Nelson [21] and a model based on a meson exchange picture, in which a kaon condensate occurs at about $4n_0$, and finally, (d) a model which allows both quark and hadron phases to be present. The composition of the star differs among these models due to differences in the nuclear interactions at high density, and, also whether or not strange baryons or strange mesons are included in the description of matter. For example, when only nucleons are included in models (a) and (b), the proton fraction in matter is determined by the density dependence of the symmetry energy; the more rapidly the symmetry energy increases with density, the greater is the proton fraction. However, neutrons remain the most abundant species in such stars. In model (b), the inclusion of hyperons, which carry strangeness, has the effect of substantially softening the EOS at high density. In particular, the presence of a substantial number of negatively charged particles, such as the Σ^- hyperon, raises the proton concentration in neutrino free matter and reduces the lepton concentrations. In model (c), K^- mesons in the condensate effectively replace electrons to achieve charge neutrality, with the result that nearly as many protons as neutrons are found in the dense interior regions of the “nucleon” star. In model (d), additional negative charge is provided by d and s quarks.

The composition of matter is significantly altered when neutrinos are trapped. This is due to the fact that at the onset of trapping, $Y_{Le} \sim 0.4$, and the electron fraction Y_e is significantly larger than that found in a cold catalyzed star. The corresponding changes in the structure are quantitatively different among the different models. However, the effects of neutrino trapping in matter containing negatively charged, strongly interacting particles, either Σ^- s in hyperonic matter, K^- s in Bose condensed matter, or quarks in matter that has undergone a phase transition at high density, are qualitatively similar. Since the size of thermal effects depends on the relative concentrations, which determine the degree to which each constituent is degenerate ($T/T_{F_i} \ll 1$, where T_{F_i} is the Fermi temperature of species i), the structural changes are expected to be accordingly different for the different

models.

4.2 Potential models

Based on a two-body potential fitted to nucleon-nucleon scattering, and a three body term whose form is suggested by theory and whose parameters are determined by the binding of few body-nuclei and the saturation properties of nuclear matter, Wiringa *et al.* [18] have performed microscopic calculations of neutron star matter at zero temperature. At high density, there are uncertainties in the three-body interactions, which are reflected in the density dependence of the symmetry energy. Calculations at finite temperature to encompass an entropy/baryon in the range $S = 1 - 2$ are not yet available. In this section, we therefore outline a schematic potential model which is designed to reproduce the results of the more microscopic calculations (see, for example, Refs. [18, 22]) of both nuclear and neutron-rich matter at zero temperature, and which can be extended to finite temperature [23].

We begin with the energy density

$$\varepsilon = \varepsilon_n^{(kin)} + \varepsilon_p^{(kin)} + V(n_n, n_p, T), \quad (7)$$

where n_n (n_p) is the neutron (proton) density and the total density $n = n_n + n_p$. The contributions arising from the kinetic parts are

$$\varepsilon_n^{(kin)} + \varepsilon_p^{(kin)} = 2 \int \frac{d^3 k}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} (f_n + f_p), \quad (8)$$

where the factor 2 denotes the spin degeneracy and f_i for $i = n, p$ are the usual Fermi-Dirac distribution functions and m is the nucleon mass. It is common to employ local contact interactions to model the nuclear potential. Such forces lead to power law density-dependent terms in $V(n)$. Since repulsive contributions that vary faster than linearly give rise to acausal behavior at high densities, care must be taken to screen such repulsive interactions [17]. Including the effect of finite-range forces between nucleons, we parametrize the potential contribution as

$$\begin{aligned} V(n_n, n_p, T) = & \frac{1}{3} A n_0 \left[\frac{3}{2} - \left(\frac{1}{2} + x_0 \right) (1 - 2x)^2 \right] u^2 \\ & + \frac{\frac{2}{3} B n_0 \left[\frac{3}{2} - \left(\frac{1}{2} + x_3 \right) (1 - 2x)^2 \right] u^{\sigma+1}}{1 + \frac{2}{3} B' n_0 \left[\frac{3}{2} - \left(\frac{1}{2} + x_3 \right) (1 - 2x)^2 \right] u^{\sigma-1}} \\ & + \frac{2}{5} u \sum_{i=1,2} \left\{ (2C_i + 4Z_i) 2 \int \frac{d^3 k}{(2\pi)^3} g(k, \Lambda_i) (f_n + f_p) \right. \\ & \left. + (C_i - 8Z_i) 2 \int \frac{d^3 k}{(2\pi)^3} g(k, \Lambda_i) [f_n(1 - x) + f_p x] \right\}, \end{aligned} \quad (9)$$

where $x = n_p/n$ and $u = n/n_0$, with n_0 denoting equilibrium nuclear matter density. The function $g(k, \Lambda_i)$ is suitably chosen to simulate finite range effects. The constants A , B , σ , C_1 , C_2 , and B' , which enter in the description of symmetric nuclear matter, and the additional constants x_0 , x_3 , Z_1 , and Z_2 , which determine the properties of asymmetric nuclear matter, are treated as parameters that are constrained by empirical knowledge.

Various limits of the energy density are of interest, and are listed below. Setting $x = 1/2$ and $f_n = f_p$, the energy density of symmetric nuclear matter is

$$\begin{aligned}\varepsilon_{nm} = & 4 \int \frac{d^3k}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} f_n + \frac{1}{2} A n_0 u^2 + \frac{B n_0 u^{\sigma+1}}{1 + B' u^{\sigma-1}} \\ & + u \sum_{i=1,2} C_i 4 \int \frac{d^3k}{(2\pi)^3} g(k, \Lambda_i) f_n ,\end{aligned}\quad (10)$$

and, with $x = 0$, the corresponding result for pure neutron matter is

$$\begin{aligned}\varepsilon_{nem} = & 2 \int \frac{d^3k}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} f_n + \frac{1}{3} A n_0 (1 - x_0) u^2 + \frac{\frac{2}{3} B n_0 (1 - x_3) u^{\sigma+1}}{1 + \frac{2}{3} B' (1 - x_3) u^{\sigma-1}} \\ & + \frac{2}{5} u \sum_{i=1,2} (3C_i - 4Z_i) 2 \int \frac{d^3k}{(2\pi)^3} g(k, \Lambda_i) f_n .\end{aligned}\quad (11)$$

At zero temperature, $f_i = \theta(k_{F_i} - k)$, where k_{F_i} is the Fermi momentum of particle i . Thus the kinetic energy densities are

$$\begin{aligned}\varepsilon_{nm}^{(kin)} &= \frac{3}{5} E_F^{(0)} n_0 u^{5/3} \quad \text{for nuclear matter} \\ \varepsilon_{nem}^{(kin)} &= 2^{2/3} \left(\frac{3}{5} E_F^{(0)} n_0 u^{5/3} \right) \quad \text{for neutron matter} ,\end{aligned}\quad (12)$$

where $E_F^{(0)} = (\hbar k_F^{(0)})^2 / 2m$ is the Fermi energy of nuclear matter at the equilibrium density. To simulate finite range effects, we investigate two commonly used functional forms for $g(k, \Lambda_i)$, which lead to closed form expressions at zero temperature.

(i) $g(k, \Lambda_i) = [1 + (k/\Lambda_i)^2]^{-1}$: In this case, the finite range terms¹ at $T = 0$ in Eq. (10) and Eq. (11) may be written as

$$V_{nm}^{(fr)} = 3n_0 u \sum_{i=1,2} C_i R_i^3 \left(\frac{u^{1/3}}{R_i} - \arctan \frac{u^{1/3}}{R_i} \right) \quad (13)$$

$$V_{nem}^{(fr)} = \frac{3}{5} n_0 u \sum_{i=1,2} (3C_i - 4Z_i) R_i^3 \left(\frac{(2u)^{1/3}}{R_i} - \arctan \frac{(2u)^{1/3}}{R_i} \right) , \quad (14)$$

¹In dynamical situations, such as heavy-ion collisions, it is more appropriate to model matter using momentum dependent Yukawa interactions, as pointed out in Ref. [24]. For static matter, both cold and hot, the simpler form chosen here is adequate insofar as identical physical properties may be recovered with suitable choices of the parameters entering the description of the EOS.

where $R_i = \Lambda_i/(\hbar k_F^{(0)})$. Note that the potential energy density for symmetric nuclear matter in Eq. (10) and Eq. (13) is the same as that employed in Ref. [17].

(ii) $g(k, \Lambda_i) = 1 - (k/\Lambda_i)^2$: Here, the finite range interactions are approximated by effective local interactions by retaining only the quadratic momentum dependence. The energy densities in Eq. (10) and Eq. (11) then take the form of Skyrme's effective interactions [25]. The finite range terms, again at $T = 0$, now read

$$\begin{aligned} V_{nm}^{(fr)} &= n_0 u^2 \sum_{i=1,2} C_i \left[1 - \frac{3}{5} \frac{u^{2/3}}{R_i^2} \right] \\ V_{nem}^{(fr)} &= \frac{2}{5} n_0 u^2 \sum_{i=1,2} (3C_i - 4Z_i) \left[1 - \frac{3}{5} \frac{(2u)^{2/3}}{R_i^2} \right]. \end{aligned} \quad (15)$$

Note that, at high density, the quadratic momentum dependence inherent in Skyrme-like (SL) interactions will eventually lead to an acausal behavior due to the $u^{8/3}$ dependence of the energy densities. This situation does not occur in case (i).

The parameters A , B , σ , C_1 , C_2 , and B' , a small parameter introduced to maintain causality, are determined from constraints provided by the empirical properties of symmetric nuclear matter at the equilibrium density $n_0 = 0.16 \text{ fm}^{-3}$. With appropriate choices of the parameters, it is possible to parametrically vary the nuclear incompressibility K_0 so that the dependence on the stiffness of the EOS may be explored. Numerical values of the parameters, appropriate for symmetric nuclear matter, are given in Table 1. EOSs based on Eqs. (7) and (9) are hereafter referred to as the BPAL EOSs if the function $g(k, \Lambda_i) = [1 + (k/\Lambda_i)^2]^{-1}$, and the SL EOSs if $g(k, \Lambda_i) = 1 - (k/\Lambda_i)^2$.

In the same vein, by suitably choosing the parameters x_0 , x_3 , Z_1 , and Z_2 , it is possible to obtain different forms for the density dependence of the symmetry energy $S(n)$ defined by the relation

$$E(n, x) = \varepsilon(n, x)/n = E(n, 1/2) + S(n)(1 - 2x)^2 + \dots, \quad (16)$$

where E is the energy per particle, and $x = n_p/n$ is the proton fraction. Inasmuch as the density dependent terms associated with powers higher than $(1 - 2x)^2$ are generally small, even down to $x = 0$, $S(n)$ adequately describes the properties of asymmetric matter. The need to explore different forms of $S(n)$ stems from the uncertain behavior at high density and has been amply detailed in earlier publications [17, 26]. We have chosen to study three cases, in which the potential part of the symmetry energy varies approximately as \sqrt{u} , u , and $2u^2/(1 + u)$, respectively, as was done in Ref. [17]. Numerical values of the parameters that generate these functional forms are given in Table 2 for the BPAL and SL EOSs. The notation $\text{BPAL}n_1n_2$ and $\text{SL}n_1n_2$ is used to denote different EOSs; n_1 refers to different values of K_0 , and $n_2 = 1, 2$ and 3 indicate, respectively, \sqrt{u} , u and $2u^2/(1 + u)$ for the dependence of the nuclear symmetry potential energy on the density.

The main advantage of casting the schematic parametrization of Ref. [17] in the form of Eq. (7) through Eq. (9) is that it is now possible to study asymmetric matter at finite temperature. As an illustration of the calculational procedure at finite temperature, consider first the case of pure neutron matter. The evaluation of the baryon density

$$n = n_n = 2 \int \frac{d^3 k}{(2\pi)^3} \left[1 + \exp \left(\frac{e_k - \mu_n}{T} \right) \right]^{-1} \quad (17)$$

requires a knowledge of the single particle spectrum

$$e_k = \frac{\hbar^2 k^2}{2m} + U(n, k; T), \quad (18)$$

where the single particle potential $U(n, k; T)$, which is explicitly momentum dependent, is obtained by a functional differentiation of the potential energy density in Eq. (11), with respect to the distribution function f_n . Explicitly,

$$U(n, k; T) = \tilde{U}(n; T) + \frac{2}{5} u \sum_{i=1,2} (3C_i - 4Z_i) \left[1 + \left(\frac{k}{\Lambda_i} \right)^2 \right]^{-2}, \quad (19)$$

for the BPAL EOS, where the explicit momentum dependence is contained in the last term. The momentum-independent part is given by

$$\begin{aligned} \tilde{U}(n; T) &= \frac{2}{3} A(1 - x_0) u \\ &+ \frac{\frac{2}{3} B(1 - x_3) u^\sigma}{\left[1 + \frac{2}{3} B'(1 - x_3) u^{\sigma-1} \right]^2} \cdot \left[(\sigma + 1) + \frac{4}{3} B'(1 - x_3) u^{\sigma-1} \right] \\ &+ \frac{2}{5n_0} \sum_{i=1,2} (3C_i - 4Z_i) 2 \int \frac{d^3 k'}{(2\pi)^3} \left[1 + \left(\frac{k'}{\Lambda_i} \right)^2 \right]^{-2} f_n(k'). \end{aligned} \quad (20)$$

For a fixed baryon density n and temperature T , Eq. (17) may be solved iteratively for the as yet unknown variable

$$\eta = \frac{\mu_n - \tilde{U}}{T}. \quad (21)$$

The knowledge of η allows the last term in Eq. (20) to be evaluated, yielding \tilde{U} , which may then be used to infer the chemical potential from

$$\mu_n = T\eta - \tilde{U}, \quad (22)$$

which is required in the calculation of the single particle spectrum e_k in Eq. (18). With this e_k , the energy density in Eq. (11) is readily evaluated. The entropy density has the same functional form as that of a non-interacting system:

$$s = -2 \int \frac{d^3 k}{(2\pi)^3} [f_n \ln f_n + (1 - f_n) \ln (1 - f_n)] , \quad (23)$$

from which the pressure is obtained using

$$P = sT + n\mu_n - \varepsilon . \quad (24)$$

The above procedure is also applicable, with obvious modifications, to a system containing unequal numbers of neutrons and protons, which is generally the case for charge-neutral matter in beta equilibrium.

4.2.1 Pure neutron matter

The influence of finite entropy on the structure of a star is most easily studied when the star is idealized to be composed of neutrons only. The top panels in Fig. 2 show the density dependence of the Landau effective mass, $m^* = k_F/(\partial e_k/\partial k)|_{k_F}$, for the BPAL22 and SL22 EOSs. For the BPAL22 model considered, the effective mass is weakly dependent on density and has a value $\simeq 0.6m$, where m is the bare mass. On the other hand, for the SL22 EOS the variation of the effective mass with density (top panels) is significantly more rapid. This qualitative difference follows from the explicit expressions for the effective mass:

$$\left(\frac{m^*}{m}\right)_{nem} = \left[1 + \sum_{i=1,2} \alpha_i u \left(1 + \frac{(2u)^{2/3}}{R_i^2} \right)^{-2} \right]^{-1} ; \quad \alpha_i = \frac{2}{5} \frac{(4Z_i - 3C_i)}{E_F^{(0)} R_i^2} \quad (25)$$

for the BPAL EOS, and

$$\left(\frac{m^*}{m}\right)_{nem} = [1 + \alpha u]^{-1} ; \quad \alpha = \alpha_1 + \alpha_2 \quad (26)$$

for the SL EOS.

Turning to the pressure (center panels of Fig. 2), it is possible to analytically establish the quadratic dependence on the entropy per baryon, and, perhaps more importantly, understand the order of magnitude of the increase in the maximum mass if we analyze the thermal contributions using the methods of Fermi liquid theory. Following the analysis in Ref. [27], the thermal pressure of interacting neutrons may be cast in the form

$$P_{th} = \left[nT \frac{\pi^2}{4} \frac{T}{T_F} \right] \cdot \frac{2}{3} \left[1 - \frac{3}{2} \frac{d \ln m^*}{d \ln n} \right] , \quad (27)$$

where the Fermi temperature, $T_F = \hbar^2 k_F^2 / 2m^*$, sets the scale of the temperature dependence of the thermodynamical functions. In the dense central regions of the star, $T/T_F \ll 1$, so the neutrons, which are the only constituents considered now, are in a highly degenerate configuration. Since, in the degenerate limit, the entropy per particle $S = (\pi^2/2)(T/T_F)$, the ratio of the thermal pressure to that of the zero temperature pressure P_0 may be expressed as

$$\frac{P_{th}}{P_0} = \left[\frac{5}{3\pi^2} S^2 + \dots \right] \left[1 - \frac{3}{2} \frac{d \ln m^*}{d \ln n} \right] \left[1 + \frac{P_{pot}}{P_{kin}} \right]^{-1}, \quad (28)$$

where P_{kin} and P_{pot} , the kinetic and potential pressures, as well as the Landau effective mass, m^* , in this relation refer to zero temperature matter. This equation adequately reproduces the exact thermal pressure at the entropies likely to be relevant in the evolution of a neutron star. The differences of Eq. (28) from the ideal gas result are obvious. First, the density dependence of the effective mass introduces a correction; this is clearly the origin of the larger thermal pressure for the SL22 case compared to the BPAL22 case for a given entropy (see Fig. 2). Second, the pressure arising from potential interactions, which is generally larger than the kinetic pressure due to the predominance of repulsive interactions at high density, produces a significant reduction in the ratio P_{th}/P_0 .

Since the thermal pressure increases quadratically with the entropy/baryon, $S = s/n$, the neutron star masses should show a similar behavior, as we see from the bottom panels of Fig. 2. Qualitatively similar results are obtained for our other EOSs with different behavior for the symmetry energy. Quantitative results for the basic physical attributes of a maximum-mass star are shown in Table 3 for the EOSs termed BPAL22 and SL22, respectively. It is clear that the maximum mass at finite entropy is well approximated by

$$M_{max}(S) = M_{max}(0) \left[1 + \lambda S^2 + \dots \right], \quad (29)$$

where the coefficient λ is EOS dependent. The values of λ given in Table 3 are quite small, $\sim 10^{-2}$.

These results highlight the point that the magnitude of the increase in the maximum mass is chiefly governed by the magnitude of the thermal pressures at fixed entropy. In the simple models considered above, the thermal pressures depend sensitively on the momentum dependence of the nuclear interactions. Insofar as one can establish this momentum dependence, either from experiment or theory, the thermal pressures may be constrained. For example, the energy dependence of the real part of the optical model potential required to explain proton-nucleus scattering experiments provides a stringent constraint on the momentum dependence of the single particle potential at nuclear density, but for values of $x \sim 0.5$. For higher densities in the range $(2 - 3)n_0$ attained in GeV/particle heavy-ion collisions, the momentum dependence largely governs the flow of matter, momentum and

energy [24]. While the BPAL single particle potentials are consistent with the observed behavior, the SL potentials, with their quadratic momentum dependence, are known to be inconsistent with data at high momentum [28]. Also, at high density, the quadratic momentum dependence has the disadvantage that it leads to an acausal behavior, due to the $u^{8/3}$ dependence of the energy densities in Eq. (15).

4.2.2 Neutrino-free matter in beta equilibrium

We turn now to the more realistic case in which matter consists of neutrons, protons, electrons, and muons, with their relative concentrations determined from the conditions of charge neutrality, Eq. (2), and equilibrium under beta decay processes in the absence of neutrino trapping, Eq. (3). The EOS of strongly interacting matter above nuclear density is given by Eq. (7) and Eq. (9). The EOS of leptons is obtained from Eq. (6).

The relative concentrations, the electron chemical potential, and the individual entropies per baryon (for $S = 1$) are shown in Fig. 3 for the BPAL22 and SL22 EOSs, both of which are characterized by a symmetry energy whose potential part varies linearly with density. Note that at high density, the proton concentration lies in the range (20 – 30)% (for these EOSs), which is balanced by an equal amount of negatively charged leptons to maintain charge neutrality. Also, the lepton contribution to the total entropy per baryon is comparable to that of the degenerate nucleons at high density, thus lowering the nucleon contribution at fixed S . This results in a smaller increase in the maximum mass than when leptons are not present, as is evident from Fig. 4, where the nucleon effective masses, isentropic pressures, and the mass curves for the corresponding EOSs are shown. The increase in the maximum mass is again quadratic with entropy. This is due to the fact that both nucleonic and leptonic pressures stem mostly from quadratic terms in the entropy, terms involving higher powers of entropy giving rather small contributions. When the nucleons are degenerate, the generalization of Eq. (28) is

$$\frac{P_{th}}{P_0} = \left[\frac{5}{3\pi^2} S^2 + \dots \right] \frac{\sum_i \frac{Y_i}{T_{F_i}} \left(1 - \frac{3}{2} \frac{d \ln m_i^*}{d \ln n_i} \right)}{\left(\sum_i \frac{Y_i}{T_{F_i}} \right)^2 (\sum_i Y_i T_{F_i})} \left[1 + \frac{P_{pot}}{P_{kin}} \right]^{-1}, \quad i = n, p \quad (30)$$

where $Y_i = n_i/n$ and $T_{F_i} = (\hbar k_{F_i})^2/(2m_i^*)$ denote the number concentration and the Fermi temperature of species i , respectively. Further, P_{kin} , P_{pot} , and $P_0 = P_{kin} + P_{pot}$ are the kinetic, potential, and the total pressures at zero temperature and include the contributions from both neutrons and protons.

Quantitative results for the physical attributes of the maximum mass star are summarized in Table 4 for the different BPAL EOSs which have varying stiffnesses and different density dependence of the symmetry energy, the latter determining the proton and lepton

concentrations in the star. Results for the SL interactions with a linear potential contribution to the symmetry energy, but for EOSs with different stiffness are shown in Table 5. The results in these tables reflect the influence of entropy on the gross properties of stars. With few exceptions, the temperature at the core remains below 100 MeV. Although the central density and pressure of the maximum mass star are significantly reduced, λ is uniformly $\sim 10^{-2}$, so the increase in the maximum mass amounts to only a few percent of the cold catalyzed star, even up to $S = 2$. In all cases studied, the increase in mass is quadratic with entropy.

The moments of inertia, I , displayed in Tables 4 and 5 refer to maximum mass stars, so that the effect of cooling a given star from entropy per baryon, S , of 2 to 0 cannot be directly assessed. We therefore show in Fig. 5 the moment of inertia as a function of the central density and of the baryonic mass, M_B . (The full dots on the various curves represent the maximum mass configurations.) Since the baryonic mass is proportional to the number of nucleons, it will remain fixed as the star cools, in the absence of accretion. Fig. 5 shows that the moment of inertia decreases as the star cools from $S = 2$ to $S = 0$. The magnitude is dependent on the mass in question, but a typical value is $\sim 20\%$. Since angular momentum is conserved, again in the absence of accretion, this implies an increase of the angular velocity of similar order. Thus a significant spin-up of the neutron star is expected to occur during the cooling process.

4.2.3 Neutrino-trapped matter in beta equilibrium

We turn now to the case in which neutrinos are trapped in matter. As mentioned earlier, we fix the electron lepton number at $Y_{Le} = 0.4$ and the muon lepton number at $Y_{L\mu} = 0$. Fig. 6 shows the effects of neutrino trapping on the relative abundance, the chemical potentials, and the partial entropies for an entropy per baryon $S = 1$. The major effect of trapping is to keep the electron concentration high so that matter is more proton rich in comparison to the case in which neutrinos are not trapped (cf. Fig. 3). The EOS with trapped neutrinos is less stiff than that without neutrinos, since the decrease in pressure due to the nuclear symmetry energy is greater than the extra leptonic pressure. This is reflected in the limiting masses obtained with neutrino trapped matter (see Table 6), which are lower than those without neutrinos (cf. Table 4). Results for the maximum mass stars are summarized in Table 6 for the BPAL models with different stiffnesses. The decrease in the maximum mass due to the effect of trapping is generally larger than the increase due to thermal effects for $S = 1$. For $S = 2$ the thermal contributions are larger, which results in a delicate balance between the two competing effects.

It is instructive to contrast the effects of thermal contributions on white dwarf and neutron star structures. Unlike the case of a white dwarf, it is not possible to analytically

predict the effects of finite entropy on the structure of a neutron star. While both configurations are degenerate, and thus one expects the dominant effects of finite entropy to enter quadratically, the role of interactions in the two cases are quite different. In contrast to the case of a white dwarf, in which the equation of state is highly ideal [13], the equation of state in a neutron star is strongly non-ideal. This results in a white dwarf having a much greater sensitivity to entropy than a neutron star. For a white dwarf, in which the electron pressure dominates, the thermal correction to the Chandrasekhar mass is about 10% at an entropy per baryon of 1 [29]. In neutron stars, the pressure support is largely provided by the strongly interacting baryons, which have relatively smaller thermal contributions to the pressure and, therefore, a smaller increase in the maximum neutron star mass. As a result, the compositional variables of the EOS play a more important role than the temperature for the structure of neutron stars.

4.3 Field theoretical models

From the results of the previous section, we notice that the central density of the maximum mass star typically exceeds $(4-5) n_0$. At such densities, the Fermi momentum and effective nucleon mass are both expected to be on the order of 500 MeV. Thus a relativistic description is preferred. Relativistic local quantum field theory models (see, for example, Ref. [20]) of finite nuclei and infinite nuclear, and neutron star, matter have had some success, albeit with rather more schematic interactions and with less sophisticated approximations than their nonrelativistic counterparts (see, for example, Refs. [18, 22]). It is our purpose here to examine the effects of finite entropy and neutrino-trapping in a relativistic description and to contrast the results with those of potential models.

Specifically, we employ a relativistic field theory model in which baryons, B , interact via the exchange of σ -, ρ -, and ω -mesons. In the case that only nucleons are considered, $B = n, p$; this is the well-known Walecka model [20], which we evaluate in the Hartree approximation (or, equivalently, at the one-loop level). It has been shown, however, that hyperons significantly soften the zero-temperature equation of state [30, 31]. Therefore, we shall also consider the case where the hyperons, Λ , Σ , and Ξ , are included in the set of baryons B . (The inclusion of the spin- $\frac{3}{2}$ Δ quartet and the Ω^- hyperon is straightforward, but does not quantitatively alter the results, since they appear at densities much higher than found in the cores of stars.) Specifically, our Lagrangian is

$$\begin{aligned} \mathcal{L}_H = & \sum_B \bar{B} (i\gamma^\mu \partial_\mu - g_{\omega B} \gamma^\mu \omega_\mu - g_{\rho B} \gamma^\mu \mathbf{b}_\mu \cdot \mathbf{t} - M_B + g_{\sigma B} \sigma) B \\ & + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - U(\sigma) \\ & - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu \\ & - \frac{1}{4} \mathbf{B}_{\mu\nu} \cdot \mathbf{B}^{\mu\nu} + \frac{1}{2} m_\rho^2 \mathbf{b}_\mu \cdot \mathbf{b}^\mu . \end{aligned} \quad (31)$$

Here the ρ -meson field is denoted by \mathbf{b}_μ , the quantity \mathbf{t} denotes the isospin operator which acts on the baryons, and the field strength tensors for the vector mesons are given by the usual expressions:— $F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, $\mathbf{B}_{\mu\nu} = \partial_\mu \mathbf{b}_\nu - \partial_\nu \mathbf{b}_\mu$.

It is straightforward to obtain the partition function for the hadronic degrees of freedom, denoted by Z_H ,

$$\begin{aligned} \ln Z_H = \beta V & \left[\frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 b_0^2 - \frac{1}{2} m_\sigma^2 \sigma^2 - U(\sigma) - \sum_B \Delta E(M_B^*) \right] \\ & + 2V \sum_B \int \frac{d^3 k}{(2\pi)^3} \ln \left(1 + e^{-\beta(E_B^* - \nu_B)} \right) . \end{aligned} \quad (32)$$

We shall consider the relativistic Hartree approximation, for which we take $U(\sigma) = 0$. We shall also consider the mean field approximation, in which the negative energy sea contributions $\Delta E(M_B^*)$ is neglected and additional scalar self-couplings are included with $U(\sigma) = (bM/3)(g_{\sigma N}\sigma)^3 + (c/4)(g_{\sigma N}\sigma)^4$. The contribution of antibaryons are not significant for the thermodynamics of interest here, and is therefore not included in Eq. (32). Here, the effective baryon masses $M_B^* = M_B - g_{\sigma B}\sigma$ and $E_B^* = \sqrt{k^2 + M_B^{*2}}$. The chemical potentials are given by

$$\mu_B = \nu_B + g_{\omega B}\omega_0 + g_{\rho B}t_{3B}b_0 , \quad (33)$$

where t_{3B} is the third component of isospin for the baryon. Note that particles with $t_{3B} = 0$, such as the Λ and Σ^0 do not couple to the ρ . When hyperons are included, the negative energy sea contribution from all baryons, inclusive of the hyperons, is considered as indicated by the notation $\Delta E(M_B^*)$.

The shift in the energy density of the negative energy baryon states is evaluated in the one loop Hartree approximation. After removing divergences, $\Delta E(M_B^*)$ can be written [32] in the form

$$\begin{aligned} \Delta E(M_B^*) = -\frac{1}{8\pi^2} & \left[4 \left(1 - \frac{\mu_r}{M} + \ln \frac{\mu_r}{M} \right) M_B(M_B - M_B^*)^3 - \ln \frac{\mu_r}{M} (M_B - M_B^*)^4 \right. \\ & + M_B^{*4} \ln \frac{M_B^*}{M_B} + M_B^3(M_B - M_B^*) - \frac{7}{2} M_B^2(M_B - M_B^*)^2 \\ & \left. + \frac{13}{3} M_B(M_B - M_B^*)^3 - \frac{25}{12} (M_B - M_B^*)^4 \right] , \end{aligned} \quad (34)$$

where $M = 939$ MeV is the nucleon mass. Here, the necessary renormalization introduces a scale parameter, μ_r . For the standard choice [20, 33] of $\mu_r/M=1$ (termed RHA), the first two terms in Eq. (34) vanish. This will not be the case for other choices of μ_r/M , which introduce explicit σ^3 and σ^4 contributions. At the phenomenological level, the σ^3 and σ^4 couplings, generated from the baryon-loop diagrams, modify the density dependence of

the energy, which makes it possible [32] to obtain nuclear matter compression moduli that are significantly lower than in the standard RHA. We shall exploit this freedom to vary μ_r/M and we call this approach the modified relativistic Hartree approximation (MRHA). In previous work [19] without hyperons, we found that while neutron star masses do not significantly constrain μ_r/M , finite nuclei favor a value of 0.79. However, our interest here is to explore the dependence on the varying stiffness of the equation of state, which is permitted by modest variations in μ_r/M , for the purpose of studying the impact on the structure of the star at finite entropy.

Using Z_H , the thermodynamic quantities can be obtained in the standard way. The pressure $P_H = TV^{-1} \ln Z_H$, the number density for species B , and the energy density ε_H are given by

$$\begin{aligned} n_B &= 2 \int \frac{d^3 k}{(2\pi)^3} \left(e^{\beta(E_B^* - \nu_B)} + 1 \right)^{-1}, \\ \varepsilon_H &= \frac{1}{2} m_\sigma^2 \sigma^2 + U(\sigma) + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 b_0^2 + \sum_B \Delta E(M_B^*) \\ &\quad + 2 \sum_B \int \frac{d^3 k}{(2\pi)^3} E_B^* \left(e^{\beta(E_B^* - \nu_B)} + 1 \right)^{-1}. \end{aligned} \quad (35)$$

The entropy density is then given by $s_H = \beta(\varepsilon_H + P_H - \sum_B \mu_B n_B)$.

The meson fields are obtained by extremization of the partition function, which yields the equations

$$\begin{aligned} m_\omega^2 \omega_0 &= \sum_B g_{\omega B} n_B \quad ; \quad m_\rho^2 b_0 = \sum_B g_{\rho B} t_{3B} n_B, \\ m_\sigma^2 \sigma &= -\frac{dU(\sigma)}{d\sigma} + \sum_B g_{\sigma B} \left\{ 2 \int \frac{d^3 k}{(2\pi)^3} \frac{M_B^*}{E_B^*} \left(e^{\beta(E_B^* - \nu_B)} + 1 \right)^{-1} + \frac{\partial}{\partial M_B^*} [\Delta E(M_B^*)] \right\}. \end{aligned} \quad (36)$$

The additional conditions needed to obtain a solution are provided by the charge neutrality requirement, Eq. (2), and, when neutrinos are not trapped, the set of chemical potential relations provided by Eq. (3). For example, when $\ell = e^-$, this implies the equalities

$$\begin{aligned} \mu_\Lambda &= \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n, \\ \mu_{\Sigma^-} &= \mu_{\Xi^-} = \mu_n + \mu_e, \\ \mu_p &= \mu_{\Sigma^+} = \mu_n - \mu_e. \end{aligned} \quad (37)$$

In the case that the neutrinos are trapped, Eq. (3) is replaced by Eq. (4). The new equalities are then obtained by the replacement $\mu_e \rightarrow \mu_e - \mu_{\nu_e}$ in the above equations. The introduction of additional variables, the neutrino chemical potentials, requires additional constraints, which we supply by fixing the lepton fractions, $Y_{L\ell}$, as noted above.

4.3.1 Neutrino-free, non-strange baryonic matter

For the MRHA calculations, the nucleon coupling constants were fitted to the equilibrium nuclear matter properties: a binding energy per particle of 16 MeV, an equilibrium density of 0.16 fm^{-3} , and a symmetry energy of 30 MeV (at the equilibrium density), as in Ref. [19]. Defining $C_i^2 = (g_{iN} M/m_i)^2$, where N represents n or p , the parameters C_ω^2 , C_σ^2 , and C_ρ^2 are reproduced in Table 7 for a range of values of μ_r/M . We see that they encompass a fairly wide range of values for the nucleon effective mass, M_N^* , and the compression modulus, K_0 , at saturation. Correspondingly, the stiffness of the equation of state shows significant variations. Note that K_0 alone is not always a reliable indicator of the stiffness at high density; for example, the equation of state for $\mu_r/M=0.73$ is noticeably stiffer than for $\mu_r/M=1.25$, yet the compression modulus is smaller, see Ref. [19].

In the mean field approximation, we use one of the parameter sets of Glendenning and Moszkowski [34], so that comparisons may be made. Specifically, we use the couplings

$$\begin{aligned} b &= 0.008659 & c &= -0.002421 \\ C_\omega^2 &= 109.14, & C_\sigma^2 &= 224.78 & \text{and} & & C_\rho^2 &= 108.49. \end{aligned} \quad (38)$$

With these constants, the equilibrium density of nuclear matter is $n_0 = 0.153 \text{ fm}^{-3}$ with the Landau effective mass $M_{N\text{sat}}^*/M = 0.827$ and compression modulus $K_0 = 240 \text{ MeV}$. This model is referred to as the GM model.

Table 8 shows the basic properties of stars in beta equilibrium for the MRHA model with various values of μ_r/M and for the GM model. The structural changes at finite entropy compared to the zero entropy case are qualitatively similar to those given by the BPAL potential model in the previous section. In most cases, λ is a little smaller in magnitude, and this is probably due to the strong repulsion of the ω -meson at high density, which increases the P_{pot} term in Eq. (28). Again, a nearly quadratic increase of the maximum mass with entropy is obtained. The central temperatures, T_c , are lower here than in the non-relativistic case. Since it is the entropy per baryon that is constant, the temperature will vary with density. The temperature profile as a function of density is shown by the full curves in the upper panel of Fig. 7 for $\mu_r/M=1.25$. The temperature is a maximum at the center of the star (here the density ratio is ~ 7 for a maximum mass star, see Table 8) and decreases with decreasing density, the fall off being particularly marked at low density. The density of a neutron star is approximately constant in the interior and drops to zero over a radial distance $\Delta R/R \sim 0.1$. Thus the interior of the star will have a constant temperature, and this will fall off rapidly in the surface region.

The top panel in Fig. 8 shows the composition of the star for the model with $\mu_r/M=1.25$. The middle panel shows the electron (or muon) chemical potential, while the bottom panel shows the individual contributions to the total entropy. Note that the lepton contribution to the entropy is smaller than in the non-relativistic case. In Fig. 9, we show in the top panel

the behavior of the Landau effective masses for neutrons and protons; these are defined by $m_n^* \equiv E_{Fn}^* = \sqrt{k_{Fn}^2 + M_n^{*2}}$, where $k_{Fn}^3 = 3\pi^2 n_n$ for neutrons, and similarly for protons. The isentropic pressures are shown in the middle panel of Fig. 9, and the bottom panel indicates the star mass versus central density ratio for fixed entropies. These results are quite similar to those shown in Figs. 3 and 4 for BPAL22, although the maximum masses obtained here are somewhat larger.

As with the potential models, the nucleonic contributions to the thermal pressure may again be simply estimated in the degenerate limit, $T/T_F \ll 1$. For one nucleon species, the Fermi temperature $T_F = k_F^2/(2E_F^*)$, and the thermal pressure for this relativistic model is given by an expression of the form (see Ref. [27] for more details)

$$P_{th} = nT \frac{\pi^2}{4} \frac{T}{T_F} \cdot \frac{1}{3} \left[1 + \left(\frac{M^*}{E_F^*} \right)^2 \left(1 - 3 \frac{d \ln M^*}{d \ln n} \right) \right] + \dots, \quad (39)$$

where the term containing the logarithmic derivative arises from the density dependence of the effective mass. The entropy per particle in the degenerate limit is again given by $S = (\pi^2/2)(T/T_F)$, but now with the Fermi temperature appropriate for a relativistic spectrum. Generalizing to two nucleon species, the total thermal pressure of the nucleons, up to quadratic terms in S , may be written as

$$P_{th} = n \left[\frac{1}{3\pi^2} S^2 + \dots \right] \frac{\sum_i \frac{Y_i}{T_{Fi}} \left[1 + \left(\frac{M_i^*}{E_{Fi}^*} \right)^2 \left(1 - 3 \frac{d \ln M_i^*}{d \ln n_i} \right) \right]}{\left(\sum_i \frac{Y_i}{T_{Fi}} \right)^2}, \quad i = n, p, \quad (40)$$

where $Y_i = n_i/n$ with n the total nucleon density. This provides an accurate approximation to the exact results.

4.3.2 Neutrino-trapped, non-strange matter

With $Y_{Le} = 0.4$ and $Y_{L\mu} = 0$, Fig. 10 shows the effects of neutrino trapping on the relative abundance, the chemical potentials, and the partial entropies for an entropy per baryon $S = 1$. The temperature profile of the star, indicated by the dotted curves in Fig. 7, differs rather little from the untrapped case. Results for the maximum mass stars are summarized in Table 9 for the MRHA and GM models. Note that the effect of trapping is similar to that found earlier with the potential model, especially for the abundance of protons due to the large concentration of electrons. The amount by which the maximum decreases, $\sim 0.07M_\odot$ when neutrinos are trapped, is also similar to the results of the potential models. Since thermal effects are smaller in the relativistic models, the maximum mass of the $S = 2$, neutrino-trapped star is always less than that of the cold $S = 0$, neutrino-free star.

Together with the findings of the earlier section, we conclude that in matter in which the only baryons are neutrons and protons, neutrino trapping usually decreases the maximum mass by a larger amount than thermal effects increase it.

4.3.3 Neutrino-free, strangeness-rich baryonic matter

When hyperons are included, their coupling constants are needed; however, these are largely unknown. We shall assume that all the hyperon coupling constants are the same as those of the Λ , for which we can take some guidance from hypernuclei. Following Glendenning and Moszkowski [34], the binding energy of the lowest Λ level in nuclear matter at saturation yields

$$\begin{aligned} B_\Lambda &= \mu_\Lambda - M_\Lambda = x_\omega g_{\omega n} \omega_0 + M_\Lambda^* - M_\Lambda, \quad \text{or} \\ -28 \text{ MeV} &= x_\omega g_{\omega n} \omega_0 - x_\sigma g_{\sigma n} \sigma, \end{aligned} \quad (41)$$

where $x_\sigma = g_{\sigma \Lambda} / g_{\sigma n}$ and $x_\omega = g_{\omega \Lambda} / g_{\omega n}$. Further, as suggested in Ref. [34] on the basis of fits to hypernuclear levels and neutron star properties, we take $x_\sigma = 0.6$. The value of x_ω may then be determined from Eq. (41). For the ρ meson, we take $x_\rho = g_{\rho \Lambda} / g_{\rho n} = x_\sigma$. The alternative choice, $x_\rho = x_\omega$, is found to produce essentially similar results.

In the MRHA model, the negative energy sea contributions $\Delta E(M_B^*)$ from all baryons, inclusive of the hyperons, contribute even when the positive energy states of the hyperons are empty. This entails a redetermination of the constants for the case in which hyperons are included. In Table 10, the coupling constants that reproduce nuclear matter saturation properties and the binding energy of the lowest Λ level in nuclear matter at saturation (with $x_\sigma = 0.6$) are given for different choices of μ_r/M . Also shown are the nucleon effective masses and the compression moduli. Since the negative energy sea contributions of all the baryons are positive, the values of C_ω^2 are somewhat smaller than those in Table 7 for the case in which only nucleons are considered. Consequently, the values of K_0 are also smaller than those in Table 7.

For the MRHA model, with $\mu_r/M=1.25$, we show in Fig. 11 the relative fractions (top panel) of the baryons and leptons in beta equilibrium, and the electron chemical potential (bottom panel) as a function of baryon density at zero temperature. One expects that Λ , with a mass of 1116 MeV, and the Σ^- , with a mass of 1197 MeV, first appear at roughly the same density, because the somewhat higher mass of the Σ^- is compensated by the presence of the electron chemical potential in the equilibrium condition (see Eq. (37)) of the Σ^- . More massive, and more positively charged, particles than these appear at higher densities. Notice that with the appearance of the negatively charged Σ^- hyperon, which competes with the leptons in maintaining charge neutrality, the lepton concentrations begin to fall. This is also reflected, for example, in the magnitude of the electron chemical

potential which saturates at around 200 MeV and begins to fall once the Σ^- population begins to rise rapidly. (That the negatively charged Σ^- is the cause for μ_e to fall with density may be verified by allowing only the neutral particles to appear. In this case, μ_e continues to rise with density, albeit with a reduced slope compared to the case in which no neutral hyperons are present.) The rapid build up of the other hyperons with increasing density has two major consequences. First, the system is strangeness-rich at high density, with nearly as many protons as neutrons. Second, since at a given total baryon density the system contains many more baryon species with sizeable concentrations, the EOS is considerably softer than when no hyperons are present. This causes the maximum mass to be reduced [30, 31].

Fig. 12 contains the corresponding results at an entropy per baryon $S = 1$. As expected, at finite temperature the hyperons attain significant fractions at lower baryon densities than at zero temperature. The order of appearance of the various hyperons follows from the chemical potential relations, Eq. (37), and their differing masses. The bottom panel of Fig. 12 shows that the baryons carry most of the entropy, since the lepton populations remain low at high density, due to the magnitude of the electron chemical potential.

Table 11 summarizes the gross features of the maximum mass star populated with hyperons. Compared to the case in which only nucleons are present, the addition of hyperons causes the central temperatures to be reduced. This is evident by comparison of the bottom and the top panels in Fig. 7. This figure also shows that with hyperons present the temperature changes rather little with density until $u < 2$, so that a constant temperature would be achieved over much of the star. The softening introduced in the MRHA EOS by hyperons is evident in the maximum masses in Table 11. These are about $0.4 - 0.9M_\odot$, smaller than the results of Table 8, for which only nucleons are allowed in matter. Notice that in some cases, the maximum mass falls below $1.44M_\odot$, and the pressure support of finite entropy is not adequate to raise the maximum mass above $1.44M_\odot$. We also give here results for the mean field GM model [34] (with $x_\sigma = x_\rho = 0.6$ and $x_\omega = 0.659$). This model shows a similar reduction in the maximum mass of $\sim 0.5 M_\odot$, due to the softening induced by hyperons. Qualitatively similar results, albeit with somewhat larger limiting masses, are obtained for the other choices of couplings in Ref. [34].

In contrast to the maximum masses of stars containing nucleons and leptons only, the maximum masses of the hyperon populated stars do not show a regular behavior with increasing entropy. In those cases for which the maximum mass increases with entropy, a quadratic increase is observed. However, with $\mu_r/M=1$, a decreasing trend with entropy is found. We have verified that this surprising behavior does not violate any laws of thermodynamics. It can be traced back to the softening of the EOS caused by the appearance of hyperons at relatively smaller densities than are found at zero temperature.

4.3.4 Neutrino-trapped, strangeness-rich matter

Fig. 13 is the counterpart of Fig. 11 for the case in which neutrinos are trapped at zero temperature. Trapped neutrinos have a large influence on the charged hyperon thresholds. For example, the appearance of the Σ^- hyperon is governed by $\mu_{\Sigma^-} = \mu_n + \mu$. Here, $\mu = \mu_e - \mu_{\nu_e}$ is much smaller than in the untrapped case for which $\mu = \mu_e$, so the appearance of the Σ^- occurs at a higher density. With the appearance of hyperons, the neutrino population begins to increase with density, in contrast to the monotonic decrease exhibited in the hyperon free case. The finite chemical potential of the neutrinos requires the electron chemical potential to be at a higher value than in the neutrino free case to maintain chemical equilibrium. Thus, unlike the neutrino free case, the electron chemical potential increases with density. Notice that muons play little role here. The preponderance of negatively charged particles, both leptons and baryons, now has the consequence that the system is proton rich over an extended region of density. These qualitative features are retained also at finite entropy with baryons carrying most of the entropy, as shown in Fig. 14.

The physical properties of the maximum mass stars with trapped neutrinos are listed in Table 12. The changes due to entropy alone are small and not always in the direction of increasing the maximum mass. Notice, however, that for each entropy shown, the maximum masses are all about $0.2M_\odot$ *larger* than those in Table 11 for neutrino free stars. Since the star has to cool down from an $S \sim 1$ configuration, with neutrinos trapped, to a configuration of $S \sim 0$ without neutrinos, the maximum stable mass decreases. This may be contrasted with the case in which no hyperons are present, in which case neutrino trapping and finite entropy effects are opposed to each other and effectively cancel. This is a general result that stems from the softening induced by the presence of negatively charged hadrons in the star, as demonstrated in the next section, where kaon condensation is allowed to occur in dense matter.

4.3.5 Metastability of neutron stars with strangeness-rich baryons

The most striking conclusion of the above discussion is the possibility of metastable neutron stars if matter contains hyperons. Metastable stars occur within a range of masses near the maximum mass of the initial configuration and remain stable only for several seconds after formation. In contrast, in matter with only nucleons, any star that is below the mass limit of the initial configuration will be stable during the subsequent evolution (in the absence of mass accretion).

In an idealized picture, the two features that govern the evolution of the maximum mass are the lepton content and finite entropy. The latter plays a minor role, but effects of the high lepton content are very important. The combined effects of lepton fraction and finite entropy are shown in Fig. 15. Here, the abscissa is the baryonic mass which is

proportional to the number of baryons in the system and is constant during the evolution of the star (in the absence of accretion of matter, again, an idealization). The ordinate is the gravitational mass, hitherto referred to simply as the mass, which includes interactions and, therefore, changes as the star evolves. If hyperons are present (lines ending with a dot), then deleptonization, that is the transition from $Y_{L_e} = 0.4$ to $Y_\nu = 0$, accompanied by heating from $S = 1$ to $S = 2$, lowers the range of baryonic masses that can be supported by the equation of state (of model GM here) from about $1.95M_\odot$ to about $1.73M_\odot$. The window in the baryonic mass in which neutron stars are metastable is thus about $0.22M_\odot$ wide. On the other hand, if hyperons are absent (lines ending with a star), the baryonic mass increases during deleptonization, and no metastability occurs. Similar results apply to the gravitational masses, which can be obtained from the baryonic masses with the help of the right panel.

Fig. 16 illustrates the evolution of the maximum mass during deleptonization. For clarity, the small effects of finite temperature are not considered here. When neutrinos diffuse out, the neutrino fraction decreases from an initial maximum value of about $Y_{\nu_e} = 0.08$, corresponding to $Y_{L_e} = 0.4$, to zero. The evolution of the neutrino fraction with time can only be obtained by solving the neutrino and energy transport equations, as was done, for example, by Burrows and Lattimer [1]. This is beyond the scope of the present work. However, if one assumes that the neutrino fraction decreases approximately uniformly with time, then one obtains a good picture of how the maximum mass will evolve. In Fig. 16, the maximum mass is shown as a function of the electron neutrino number per baryon, Y_{ν_e} , for the GM model. It is evident that different compositions lead to different trends. In nucleonic matter, the maximum mass slowly *increases* as neutrinos diffuse out; in hyperonic matter, maximum mass *decreases*. It is also clear that the rate of change is largest shortly before the neutrino fraction drops to zero. Similar qualitative behavior is obtained for other choices of mean field parameters from Ref. [34].

4.4 Matter with kaon condensation

The idea that, above some critical density, the ground state of baryonic matter might contain a Bose-Einstein condensate of kaons is due to Kaplan and Nelson [21]. The formulation, in terms of chiral perturbation theory, was subsequently discussed by Politzer and Wise [35] and Brown *et al.* [36]. The composition and structure of kaon condensed stars and also some evolutionary aspects were considered by Thorsson, Prakash and Lattimer [6]. Further related calculations at the mean-field level may be found in Refs. [37, 38]. Most recently, loop contributions have also been investigated [39]. Depending on the parameters employed, particularly the strangeness content of the proton, kaon condensation is typically found to occur at about 4 times the equilibrium nuclear matter density. This has the effect of softening the equation of state and lowering the maximum mass of the neutron star.

In chiral models the kaon-nucleon interaction occurs directly via four point vertices; however, it can also be modelled as an indirect interaction that arises from the exchange of mesons [40]. The latter approach has the virtue that it is more consistent with the meson exchange picture that is usually employed for the baryon interactions, and it is of interest to compare the predictions with those of the chiral model. A further question is the role that hyperons might play in addition to kaons, since we have seen that they appear at a density similar to, or somewhat lower than, the condensate threshold. This raises the issue of the interplay between the strange baryons and mesons and the net strangeness content of neutron stars. These questions have been the subject of recent work [41, 42, 43], and we shall address them in the present context.

Our purpose here is to determine the impact of a kaon condensate on protoneutron stars. In order to keep the number of calculations within bounds, we will focus on the mean field case with the GM parameters. We will study the effect of neutrino trapping in chiral and meson exchange models, both with and without hyperons. As regards finite entropy effects, we have seen that they are less significant than neutrino trapping. It is not clear how to develop a consistent finite temperature formalism for the chiral case, while it is reasonably straightforward for the meson exchange model. We therefore study the effects of finite entropy in the latter approach and restrict our chiral formalism to $T = 0$.

4.4.1 Chiral formalism

The Kaplan-Nelson $SU(3) \times SU(3)$ chiral Lagrangian for the kaons and the s -wave kaon-baryon interactions takes the form

$$\mathcal{L}_K = \frac{1}{4}f^2 \text{Tr} \partial_\mu U \partial^\mu U + C \text{Tr} m_q (U + U^\dagger - 2) + i \text{Tr} \bar{B} \gamma^\mu [V_\mu, B] + a_1 \text{Tr} \bar{B} (\xi m_q \xi + h.c.) B + a_2 \text{Tr} \bar{B} B (\xi m_q \xi + h.c.) + a_3 \{ \text{Tr} \bar{B} B \} \{ \text{Tr} (m_q U + h.c.) \} . \quad (42)$$

Here, $U = \xi^2$ is the non-linear field involving the pseudoscalar meson octet, from which we retain only the K^\pm contributions—

$$U = \exp \left(\frac{\sqrt{2}i}{f} M \right) \quad ; \quad M = \begin{pmatrix} 0 & 0 & K^+ \\ 0 & 0 & 0 \\ K^- & 0 & 0 \end{pmatrix} . \quad (43)$$

The baryon octet – nucleons plus hyperons – is given by

$$B = \begin{pmatrix} \sqrt{\frac{1}{2}} \Sigma^0 + \sqrt{\frac{1}{6}} \Lambda & \Sigma^+ & p \\ \Sigma^- & -\sqrt{\frac{1}{2}} \Sigma^0 + \sqrt{\frac{1}{6}} \Lambda & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}} \Lambda \end{pmatrix} . \quad (44)$$

In Eq. (42), the quark mass matrix $m_q = \text{diag}(0, 0, m_s)$; *i.e.*, only the mass of the strange quark is taken to be non-zero. For the mesonic vector current, V_μ , only the time component survives in an infinite system with $V_0 = \frac{1}{2}(\xi^\dagger \partial_0 \xi + \xi \partial_0 \xi^\dagger)$. Also, the pion decay constant $f = 93$ MeV, and C, a_1, a_2 and a_3 are constants. After some algebra, the relevant part of \mathcal{L}_K takes the form

$$\begin{aligned} \mathcal{L}_K = & \left(\frac{\sin \chi}{\chi} \right)^2 \left\{ \partial_\mu K^+ \partial^\mu K^- + \frac{i}{4f^2} \frac{(K^+ \partial_0 K^- - K^- \partial_0 K^+)}{\cos^2 \frac{1}{2}\chi} \sum_B (Y_B + q_B) B^\dagger B \right. \\ & - \left(m_K^2 + \frac{m_s}{2f^2} \sum_B [(a_1 + a_2)(1 + Y_B q_B) + (a_1 - a_2)(q_B - Y_B) + 4a_3] \bar{B}B \right. \\ & \left. \left. + \frac{m_s}{6f^2} (a_1 + a_2)(2\bar{\Lambda}\Lambda + \sqrt{3}[\bar{\Sigma}^0\Lambda + \bar{\Lambda}\Sigma^0]) \right) \frac{K^+ K^-}{\cos^2 \frac{1}{2}\chi} \right\}, \end{aligned} \quad (45)$$

where q_B and Y_B are the baryon charge and hypercharge, respectively, and $q_B = \frac{1}{2}Y_B + t_{3B}$. (The hypercharge is given in terms of the baryon number and strangeness by $Y_B = b_B + S_B$.) In Eq. (45), we have defined $\chi^2 = 2K^+K^-/f^2$ and taken the kaon mass to be given by $m_K^2 = 2Cm_s/f^2$. We have not included in Eq. (45) terms which simply give a constant shift to the baryon masses; they indicate that $a_1 m_s = -67$ MeV and $a_2 m_s = 134$ MeV, using the hyperon-nucleon mass differences. The remaining constant $a_3 m_s$ is not accurately known, and we shall use values in the range -134 to -310 MeV corresponding to 0 to 20% strangeness content for the proton. The corresponding range for the kaon-nucleon sigma term,

$$\Sigma^{KN} = -\frac{1}{2}(a_1 + 2a_2 + 4a_3)m_s, \quad (46)$$

is 167–520 MeV. Some guidance is provided by recent lattice gauge simulations [44], which find that the strange quark condensate in the nucleon is large, *i.e.*, $\langle N|\bar{s}s|N \rangle = 1.16 \pm 0.54$. From the relation $m_s \langle \bar{s}s \rangle_p = -2(a_2 + a_3)m_s$ and using $m_s = 150$ MeV, we obtain $a_3 m_s = -(220 \pm 40)$ MeV, which is in the middle of our range of values.

The pure kaon part of Eq. (45) gives a contribution to the grand potential, $\Omega_K = -T \ln Z_K$, which can be evaluated in the mean field approximation by writing the time dependence of the fields $K^\pm = \frac{1}{\sqrt{2}}f\theta e^{\pm i\mu t}$ [6]. Here, θ gives the condensate amplitude, and μ is the kaon chemical potential. One easily finds

$$\Omega_K = Vf^2(2m_K^2 \sin^2 \frac{1}{2}\theta - \frac{1}{2}\mu^2 \sin^2 \theta). \quad (47)$$

The kaon-baryon interactions in Eq. (45) can be incorporated in the baryon Lagrangian of Eq. (31) by suitable definitions of the effective masses and chemical potentials. Notice first that the $\Lambda - \Sigma^0$ mass matrix needs to be diagonalized. We write it in the form

$$\begin{pmatrix} 2\alpha & 2\beta \\ 2\beta & 2\gamma \end{pmatrix}, \quad \text{where}$$

$$\begin{aligned}
2\alpha &= M_\Lambda - g_{\sigma\Lambda}\sigma + (\frac{5}{3}a_1 + \frac{5}{3}a_2 + 4a_3)m_s \sin^2 \frac{1}{2}\theta \\
2\beta &= 3^{-\frac{1}{2}}(a_1 + a_2)m_s \sin^2 \frac{1}{2}\theta \\
2\gamma &= M_\Sigma - g_{\sigma\Sigma}\sigma + (a_1 + a_2 + 4a_3)m_s \sin^2 \frac{1}{2}\theta .
\end{aligned} \tag{48}$$

Then it is straightforward to obtain the eigenstates and the masses

$$\begin{aligned}
H_1 &= \frac{\Sigma^0 - \delta\Lambda}{(1 + \delta^2)^{\frac{1}{2}}} ; \quad H_2 = \frac{\Lambda + \delta\Sigma^0}{(1 + \delta^2)^{\frac{1}{2}}} \quad \text{with} \quad 2\beta\delta = \gamma - \alpha - \sqrt{(\gamma - \alpha)^2 + 4\beta^2} \\
M_{H_1}^* &= \gamma + \alpha + \sqrt{(\gamma - \alpha)^2 + 4\beta^2} ; \quad M_{H_2}^* = \gamma + \alpha - \sqrt{(\gamma - \alpha)^2 + 4\beta^2} .
\end{aligned} \tag{49}$$

Since $\gamma > \alpha$ here, in the limit of no mixing ($\beta = 0$) δ is zero. Henceforth, the sum over baryon states, B , includes H_1 and H_2 along with the n , p , $\Sigma^{+,-}$, $\Xi^{0,-}$. For the latter cases, the masses are given by

$$M_B^* = M_B - g_{\sigma B}\sigma + [(a_1 + a_2)(1 + Y_B q_B) + (a_1 - a_2)(q_B - Y_B) + 4a_3]m_s \sin^2 \frac{1}{2}\theta . \tag{50}$$

The chemical potentials μ_B are given in terms of the effective chemical potentials, ν_B , by

$$\mu_B = \nu_B + g_{\omega B}\omega_0 + g_{\rho B}t_{3B}b_0 - (Y_B + q_B)\mu \sin^2 \frac{1}{2}\theta . \tag{51}$$

Then, the zero-temperature limit of Eq. (32) yields the baryon grand potential $\Omega_H = -T \ln Z_H$. (We consider only the mean field case here, so $\Delta E = 0$.)

The total hadron grand potential $\Omega_{\text{tot}} = \Omega_H + \Omega_K = -PV$, where P is the total pressure. The energy density is given by $\varepsilon = -P + \sum_B \mu_B n_B + \mu n_K$, where the baryon number density, n_B , is given by the zero-temperature limit of Eq. (35) and the kaon number density is

$$n_K = -\frac{1}{V} \frac{\partial \Omega_{\text{tot}}}{\partial \mu} = f^2(\mu \sin^2 \theta + 4b \sin^2 \frac{1}{2}\theta) \quad \text{with} \quad b = \sum_B (Y_B + q_B)n_B/(4f^2) . \tag{52}$$

The meson fields are obtained by extremizing Ω_{tot} , yielding Eq. (36) with $T = 0$. The condensate amplitude, θ , is also found by extremizing Ω_{tot} . This yields the solutions $\theta = 0$ (no condensate), or, if a condensate exists, the equation

$$\begin{aligned}
\mu^2 \cos \theta + 2\mu b - m_K^2 - d_1 - d_2 &= 0 \quad \text{where} \\
2f^2 d_1 &= \sum_{B \neq H_1, H_2} [(a_1 + a_2)(1 + Y_B q_B) + (a_1 - a_2)(q_B - Y_B) + 4a_3]m_s n_B^s , \\
2f^2 d_2 \sin^2 \frac{1}{2}\theta &= \sum_{B=H_1, H_2} (M_B^* + g_{\sigma B}\sigma)n_B^s - M_\Lambda n_{H_1}^s - M_\Sigma n_{H_2}^s \\
&\quad + \frac{1}{1 + \delta^2}(M_\Lambda - M_\Sigma)(n_{H_1}^s - n_{H_2}^s) .
\end{aligned} \tag{53}$$

Here, we have taken $g_{\sigma\Lambda} = g_{\sigma\Sigma}$ for simplicity, and the baryon scalar density is

$$n_B^s = \frac{1}{\pi^2} \int_0^{k_{FB}} dk k^2 \frac{M_B^*}{(k^2 + M_B^{*2})^{\frac{1}{2}}} , \quad (54)$$

with k_{FB} denoting the baryon Fermi momentum. Equation (53) is equivalent to the requirement that μ be equal to the energy of the K^- zero-momentum state.

Finally, we need to satisfy the charge neutrality condition of Eq. (2), which reads

$$\sum_B q_B n_B - n_K - n_e - n_\mu = 0 , \quad (55)$$

and the chemical equilibrium conditions of Eq. (37). In the latter, we replace μ_{Σ^0} and μ_Λ by μ_{H_1} and μ_{H_2} ; thus, the first of Eqs. (37) becomes $\mu_{H_1} = \mu_{H_2} = \mu_{\Xi^0} = \mu_n$. Also, chemical equilibrium in the reaction $n \leftrightarrow p + K^-$ requires that the kaon chemical potential satisfy $\mu = \mu_n - \mu_p$.

We shall also need the optical potential for a kaon in nuclear matter for comparison with the meson exchange approach. We can use Eq. (45) with equal numbers of neutrons and protons, no hyperons, and no condensate ($\chi = 0$). Lagrange's equations for an s -wave K^- , with $K^- = k^-(\mathbf{x})e^{-iEt}$ and $E = \sqrt{p^2 + m_K^2}$, can be written

$$\begin{aligned} [\nabla^2 + E^2 - m_K^2]k^-(\mathbf{x}) &= \left[-\frac{3nE}{4f^2} - \frac{\Sigma^{KN}n^s}{f^2} \right] k^-(\mathbf{x}) \\ &= 2m_K U_{opt}^K k^-(\mathbf{x}) , \end{aligned} \quad (56)$$

where n is the density of nuclear matter and n^s is the scalar density. For a zero-momentum K^- meson, the optical potential reduces to

$$U_{opt}^K = S_{opt}^K + V_{opt}^K \quad ; \quad S_{opt}^K = -\frac{\Sigma^{KN}n^s}{2m_K f^2} \quad ; \quad V_{opt}^K = -\frac{3n}{8f^2} . \quad (57)$$

Equilibrium nuclear matter density fixes $V_{opt}^K = -51$ MeV, and, for our range of values of the sigma term, $S_{opt}^K = -(22-69)$ MeV. Thus, $U_{opt}^K \sim -100$ MeV. This is about half of the favored “deep solution” obtained by Friedman, Gal and Batty [45] in their analysis of the kaonic atom data, although it is comparable to the value obtained in their $t_{eff}\rho$ approximation. There are, of course, uncertainties in the analysis and also in simply expropriating the real part of a complex potential.

4.4.2 Meson exchange formalism

In this approach, we take a Lagrangian for the kaon sector, which contains the usual kinetic energy and mass terms, along with the meson interactions,

$$\mathcal{L}_K = \partial_\mu K^+ \partial^\mu K^- - (m_K^2 - g_{\sigma K} m_K \sigma) K^+ K^- + i [g_{\omega K} \omega^\mu + g_{\rho K} b^\mu] (K^+ \partial_\mu K^- - K^- \partial_\mu K^+). \quad (58)$$

Here, b^μ denotes the ρ^0 field. (The vacuum kaon mass, m_K , is present in the third term to render $g_{\sigma K}$ dimensionless.) Schaffner and Mishustin [43] have included an additional four-point interaction in their Lagrangian, so that Lagrange's equations yield $\partial_\mu \omega^\mu = 0$ as required for a particle of spin 1 [46]. We would argue that at the mean field level the vector fields are constants, so the divergence is necessarily zero, and Eq. (58) is sufficient. We can simplify notation by introducing an effective kaon mass defined by

$$m_K^{*2} = m_K^2 - g_{\sigma K} m_K \sigma. \quad (59)$$

Since only the time components of the vector fields survive, it is also useful to define

$$X = g_{\omega K} \omega_0 + g_{\rho K} b_0. \quad (60)$$

In order to determine the kaon partition function at finite temperature, we generalize the procedure outlined in Kapusta [13]. First, by studying the invariance of the Lagrangian under the transformation $K^\pm \rightarrow K^\pm e^{\pm i\alpha}$, the conserved current density can be identified. The zero component, *i.e.* the charge density, is

$$J_0 = i(K^+ \partial_0 K^- - K^- \partial_0 K^+) + 2X K^+ K^-. \quad (61)$$

Next, we transform to real fields ϕ_1 and ϕ_2 ,

$$K^\pm = (\phi_1 \pm \phi_2)/\sqrt{2}, \quad (62)$$

and determine the conjugate momenta

$$\pi_1 = \partial_0 \phi_1 - X \phi_2 \quad ; \quad \pi_2 = \partial_0 \phi_2 + X \phi_1. \quad (63)$$

The Hamiltonian density is $\mathcal{H}_K = \pi_1 \partial_0 \phi_1 + \pi_2 \partial_0 \phi_2 - \mathcal{L}_K$, and the partition function of the grand canonical ensemble can then be written as the functional integral

$$Z_K = \int [d\pi_1][d\pi_2] \int_{\text{periodic}} [d\phi_1][d\phi_2] \times \exp \left\{ \int_0^\beta d\tau \int d^3x \left(i\pi_1 \frac{\partial \phi_1}{\partial \tau} + i\pi_2 \frac{\partial \phi_2}{\partial \tau} - \mathcal{H}_K(\phi_i, \pi_i) + \mu J_0(\phi_i, \pi_i) \right) \right\}. \quad (64)$$

Here, μ is the chemical potential associated with the conserved charge density, and the fields obey periodic boundary conditions in the imaginary time $\tau = it$, namely $\phi_i(\mathbf{x}, 0) = \phi_i(\mathbf{x}, \beta)$.

The Gaussian integral over momenta in Eq. (64) is easily performed. Next the fields are Fourier decomposed according to

$$\begin{aligned}\phi_1 &= f\theta \cos \alpha + \sqrt{\frac{\beta}{V}} \sum_{n,\mathbf{p}} e^{i(\mathbf{p} \cdot \mathbf{x} + \omega_n \tau)} \phi_{1,n}(\mathbf{p}) \\ \phi_2 &= f\theta \sin \alpha + \sqrt{\frac{\beta}{V}} \sum_{n,\mathbf{p}} e^{i(\mathbf{p} \cdot \mathbf{x} + \omega_n \tau)} \phi_{2,n}(\mathbf{p}) ,\end{aligned}\quad (65)$$

where the first term describes the condensate, so that in the second term $\phi_{1,0}(\mathbf{p} = 0) = \phi_{2,0}(\mathbf{p} = 0) = 0$. The Matsubara frequency $\omega_n = 2\pi nT$. The partition function can then be written

$$\begin{aligned}Z_K &= N \int \prod_{n,\mathbf{p}} [d\phi_{1,n}(\mathbf{p})][d\phi_{2,n}(\mathbf{p})] e^S , \quad \text{where} \\ S &= \frac{1}{2}\beta V f^2 \theta^2 (\mu^2 + 2\mu X - m_K^{*2}) - \frac{1}{2} \sum_{n,\mathbf{p}} (\phi_{1,-n}(-\mathbf{p}), \phi_{2,-n}(-\mathbf{p})) \mathbf{D} \begin{pmatrix} \phi_{1,n}(\mathbf{p}) \\ \phi_{2,n}(\mathbf{p}) \end{pmatrix} , \\ \mathbf{D} &= \beta^2 \begin{pmatrix} \omega_n^2 + p^2 + m_K^{*2} - 2\mu X - \mu^2 & 2(\mu + X)\omega_n \\ -2(\mu + X)\omega_n & \omega_n^2 + p^2 + m_K^{*2} - 2\mu X - \mu^2 \end{pmatrix} .\end{aligned}\quad (66)$$

We define the K^\pm energies according to

$$\omega^\pm(p) = \sqrt{p^2 + m_K^{*2} + X^2} \pm X , \quad (67)$$

and, in most cases, we shall suppress the explicit dependence of ω^\pm on p . With this definition, the determinant of \mathbf{D} is

$$\det \mathbf{D} = \beta^4 [\omega_n^2 + (\omega^- - \mu)^2] [\omega_n^2 + (\omega^+ + \mu)^2] . \quad (68)$$

Then

$$\ln Z_K = \frac{1}{2}\beta V f^2 \theta^2 (\mu^2 + 2\mu X - m_K^{*2}) - \frac{1}{2} \sum_{n,\mathbf{p}} \ln \det \mathbf{D} . \quad (69)$$

Performing the sum over n and neglecting the zero-point contribution, which is not appropriate to a mean field approach, we obtain

$$\begin{aligned}\ln Z_K &= \frac{1}{2}\beta V f^2 \theta^2 (\mu^2 + 2\mu X - m_K^{*2}) \\ &\quad - V \int_0^\infty \frac{d^3 p}{(2\pi)^3} [\ln(1 - e^{-\beta(\omega^- - \mu)}) + \ln(1 - e^{-\beta(\omega^+ + \mu)})] .\end{aligned}\quad (70)$$

The partition function for the baryon sector takes precisely the form given in Eq. (32), and the total partition function is $Z_{\text{total}} = Z_H Z_K$. Extremization of Z_{total} yields the fields. It is useful to define the thermal quantities

$$\begin{aligned} n_K^{TH} &= \int \frac{d^3 p}{(2\pi)^3} [f_B(\omega^- - \mu) - f_B(\omega^+ + \mu)] , \\ A_K^{TH} &= \int \frac{d^3 p}{(2\pi)^3} (p^2 + m_K^{*2} + X^2)^{-\frac{1}{2}} [f_B(\omega^- - \mu) + f_B(\omega^+ + \mu)] , \end{aligned} \quad (71)$$

where the Bose occupation probability $f_B(x) = (e^{\beta x} - 1)^{-1}$. Then the field equations are

$$\begin{aligned} m_\omega^2 \omega_0 &= \sum_B g_{\omega B} n_B - g_{\omega K} \left[(f\theta)^2 \mu + n_K^{TH} - X A_K^{TH} \right] \\ m_\rho^2 b_0 &= \sum_B g_{\rho B} t_{3B} n_B - g_{\rho K} \left[(f\theta)^2 \mu + n_K^{TH} - X A_K^{TH} \right] \\ m_\sigma^2 \sigma &= -\frac{dU(\sigma)}{d\sigma} + 2 \sum_B g_{\sigma B} \int \frac{d^3 k}{(2\pi)^3} \frac{M_B^*}{E_B^*} \left(e^{\beta(E_B^* - \nu_B)} + 1 \right)^{-1} \\ &\quad + \frac{1}{2} g_{\sigma K} m_K \left[(f\theta)^2 + A_K^{TH} \right] . \end{aligned} \quad (72)$$

Notice that the condensate contributes directly to the equations of motion (72), whereas in chiral models the contribution appears in the effective chemical potentials and effective masses. The condensate amplitude, θ , is also found by extremization of Z_{total} . This yields the solutions $\theta = 0$ (no condensate), or, if a condensate exists, the equation

$$\mu^2 + 2\mu X - m_K^{*2} = [\mu - \omega^-(0)][\mu + \omega^+(0)] = 0 . \quad (73)$$

Since μ is positive here, we only have the possibility of a K^- condensate when $\mu = \omega^-(0)$. We illustrate the behavior of the kaon energies with density in Fig. 17. As the chemical potential μ increases, a condensate forms when μ becomes equal to $\omega^-(0)$. At higher densities, a condensate is still present so that μ remains equal to $\omega^-(0)$. Whether the $\omega^+(0)$ energy increases rapidly with density, as in Fig. 17, or remains more nearly constant, depends on the strength of the couplings employed. Notice that, utilizing Eq. (72) in Eq. (73), one obtains a threshold (θ infinitesimal) equation resembling Eq. (53) of the chiral case; although the weightings of the various baryons are different, and the $dU/d\sigma$ term does not play a role in the chiral case (see the further discussion below). Above threshold, θ enters in different ways in the two models.

The baryon thermodynamic variables are given by Eq. (35). For the kaons, the partition function, Eq. (70), gives the pressure, $P_K = TV^{-1} \ln Z_K$; notice that Eq. (73) indicates

that the condensate gives zero contribution to the pressure. The kaon number density is easily found to be

$$n_K = f^2 \theta^2 (\mu + X) + n_K^{TH}. \quad (74)$$

If we write the baryon energy density in the form of Eq. (35), then, after using the equations of motion Eqs. (72) and (73), the remaining part of the energy density, which arises from the kaons, can be written

$$\varepsilon_K = (f \theta m_K^*)^2 + \int \frac{d^3 p}{(2\pi)^3} [\omega^-(p) f_B(\omega^- - \mu) + \omega^+(p) f_B(\omega^+ + \mu)] + X[n_K^{TH} - X A_K^{TH}]. \quad (75)$$

The total entropy density can be obtained from the standard thermodynamic identity

$$s = \beta(\varepsilon + P - \mu n_K - \sum_B \mu_B n_B). \quad (76)$$

The above equations can be applied at zero temperature, in which case the thermal Bose occupation probabilities are zero, and the Fermi occupation probabilities become step functions cut off at the Fermi momentum.

As before, the neutron star must be charge neutral, *i.e.*

$$\sum_B q_B n_B - n_K - n_e - n_\mu = 0, \quad (77)$$

and in chemical equilibrium, with Eq. (37) satisfied and $\mu = \mu_n - \mu_p$.

Finally, as in the chiral model, we can determine the value of the optical potential felt by a single kaon at zero momentum. The analogue of Eq. (56) gives

$$U_{opt}^K \equiv \mathcal{S}_{opt}^K + \mathcal{V}_{opt}^K \quad ; \quad \mathcal{S}_{opt}^K = -\frac{1}{2} g_{\sigma K} \sigma \quad ; \quad \mathcal{V}_{opt}^K = -g_{\omega K} \omega_0. \quad (78)$$

Since we want to compare the chiral and meson exchange approaches, we will demand that they yield the same optical potential in nuclear matter. This should be a reasonable way of ensuring that the parameterizations are compatible. Thus, we choose \mathcal{S}_{opt}^K and \mathcal{V}_{opt}^K to be equal to S_{opt}^K and V_{opt}^K from Eq. (57). For the as yet undetermined kaon-rho meson coupling, we take $g_{\rho K}/g_{\rho N} = 1/3$, as suggested by naive quark counting.

4.4.3 Kaon condensation in non-strange baryonic matter

Let us begin by studying the critical, or threshold, density at which kaons start to condense. The equations obtained in both the chiral and the meson-exchange models can be written in the form

$$\mu^2 + 2\mu\alpha - m_K^{*2} = 0. \quad (79)$$

For the present, the only baryons we consider are nucleons, in which case the chiral model expressions for α and m_K^{*2} are

$$\alpha = \frac{2n_p + n_n}{2f^2}$$

$$m_K^{*2} = m_K^2 + \left[2a_1 n_p^s + (2a_2 + 4a_3)(n_p^s + n_n^s) \right] \frac{m_s}{2f^2}, \quad (80)$$

respectively, in nucleons-only matter. In the meson-exchange model, at zero temperature, we have

$$\alpha = (G_{KN}^\omega - \frac{1}{2}G_{KN}^\rho)n_n + (G_{KN}^\omega + \frac{1}{2}G_{KN}^\rho)n_p$$

$$m_K^{*2} = m_K^2 + G_{KN}^\sigma m_K \left[\frac{1}{g_{\sigma N}} \frac{dU(\sigma)}{d\sigma} - n_n^s - n_p^s \right], \quad (81)$$

where we have used the definitions $G_{KN}^i = g_{iN} g_{iK} / m_i^2$, with $i = \sigma, \omega$ and ρ . If we specialize to isospin symmetric nuclear matter with $\mu = \mu_n - \mu_p = \mu_e = 0$, Eqs. (80) and (81) take the form

$$m_K^2 f^2 = n^s \Sigma^{KN} \quad \text{and}$$

$$m_K^2 = G_{KN}^\sigma \left[n^s - \frac{1}{g_{\sigma N}} \frac{dU(\sigma)}{d\sigma} \right], \quad (82)$$

respectively. These results give the critical density for condensation in symmetric nuclear matter to be $u \geq 6.5$, well in excess of the values of u_{crit} shown in Table 13 for neutrino-free matter in beta equilibrium. Schaffner et al. [47] have recently emphasized this and shown that, depending on the chosen parameters, condensation may not occur at all in *nuclear* matter.

The presence of leptons in stellar matter lowers the critical density for condensation by a significant amount. In Table 13, we list the critical density for kaon condensation at $T = 0$ for three choices of the constant $a_3 m_s$, and for matter without and with hyperons. (The latter case will be discussed in the next subsection.) Results shown include the neutrino-free and trapped neutrino cases. In all cases shown here, baryons are described using the GM model, but kaons are described using both the chiral and meson exchange models.

Consider first the zero temperature case, in which neutrinos are absent. The critical densities show a marked reduction as the magnitude of $a_3 m_s$ increases, since this enhances the interactions; but they are not very sensitive to the choice of the model used to describe kaonic interactions (*i.e.*, chiral versus the meson exchange model) as long as compatibility of the kaon optical potentials is required. The mean field model, GM, yields somewhat lower values for the critical density than the MRHA models (see Ref. [41]), due to the more rapid increase of the scalar fields that enter the interaction terms of Eqs. (80) and (81).

We remark that if we were to replace the scalar density in these calculations by the number density, the critical density ratio, u_{crit} , for condensation at $T = 0$ drops by approximately 1 unit in the density ratio u . (The effect appears to be of smaller magnitude in the recent work of Maruyama et al. [38] employing mean field theory.)

For neutrino-free matter, the effects of finite temperature on the onset of kaon condensation are shown in Fig. 18. (The meson exchange formalism is used for the finite temperature results in this subsection.) The results shown are for the GM model and for three different values of Σ^{KN} . The proton and kaon concentrations in the top two panels, and the chemical potential μ in the third panel, refer to the values at the critical density ratio u_{crit} shown in the bottom panel. Thermal effects give rise to a non-negligible net negative kaon concentration, which results in larger proton concentrations than those for the zero entropy case. This hinders the onset of condensation. However, the compensating changes in the chemical potential μ with temperature (in the range that supports an entropy per baryon up to 2) result in only small net changes in u_{crit} for condensation. Similar results are obtained in the MRHA models. Thus, the effects of condensation remain significant, even at finite entropy.

In Fig. 19, the relative concentrations, the electron chemical potential, and the hadronic and leptonic contributions to the entropy are shown for $S = 1$ for the case of neutrino-free matter. The results are for the GM model with $a_3 m_s = -222$ MeV. At finite temperatures and at densities below that for condensation, the kaon concentration remains smaller than that of the other particles; consequently, the changes induced in the total pressure and energy density are small. As in the case of zero temperature, the kaon concentration builds up rapidly for densities above the condensation density, which results in a softening of the equation of state. For an entropy per baryon up to 2, the relative concentrations of the various particles essentially retain the zero temperature behavior. Thus, thermal effects do not change drastically the softening induced by the condensation. The changes in the structure of the star, in particular, the changes in the maximum mass, are therefore at the few per cent level, as in the case of stars without condensation.

We turn now to the effects of trapped neutrinos. The results for this case are contained in Table 13 and Figs. 20 and 21. Due to the behavior of the chemical potential (which is similar to that shown in Figs. 6 and 10), the critical density for kaon condensation is much higher when neutrinos are trapped than in the case of neutrinos having left the star. (Compare the critical densities with the central densities in Table 9.) Thus, in the trapped case, the hadronic pressures are relatively larger for a substantial range of densities than in the neutrino-free case.

In Table 14, we give the stellar properties for the neutrino-free and neutrino-trapped cases. Comparison of the neutrino-free results with Table 8 shows rather small effects, except for the largest magnitudes for the parameter $a_3 m_s$. It is only for these cases that the central densities significantly exceed the critical densities (Table 13) and allow a size-

able core of condensed kaons to appear. This also occurs in the MRHA models for the larger magnitudes of $a_3 m_s$. Turning to the trapped case, we see that the maximum mass is generally larger. This can be contrasted with Tables 8 and 9, where K^- particles were absent; and the maximum mass was a little less in the trapped case. This qualitative change engendered by kaons is similar to that previously noted for hyperons.

4.4.4 Kaon condensation in strangeness-rich baryonic matter

We have affirmed the importance of hyperons in neutron stars, so it is interesting to see how they affect the phenomenon of kaon condensation [41]. The necessary formalism has been outlined above. Since finite temperature effects are not too large, we focus on the zero temperature case. For the chiral model with $a_3 m_s = -222$ MeV in conjunction with the mean field GM description of the baryons, the results are shown in Fig. 22. The critical densities are given in Table 13. These critical densities are higher than those in the case in which hyperons were absent. The reason is clear. Once a significant number of negatively charged hyperons are present (panel (a) of Fig. 20), the electron chemical potential, μ , begins to decrease with density (panel (b)). Since kaon condensation occurs when this same chemical potential equals the energy of the zero-momentum K^- state, it is necessary to go to higher density where the interactions are able reduce the energy further. For the MRHA, in almost all cases, the critical density is beyond the central densities in Table 11, so that condensation does not occur. On the other hand, it can take place in the mean field model, GM, provided the magnitude of $a_3 m_s$ is not too small. This follows from the fact that the scalar densities increase more rapidly in mean field models than in the MRHA, which affects the interaction terms d_1 and d_2 in Eqs. (53). Some insight into the role of the scalar densities may be gained by examining the threshold condition in the chiral approach when only Σ^- and Λ hyperons are present:

$$\begin{aligned} \mu^2 + \frac{(2n_p + n_n - n_{\Sigma^-})}{2f^2} \mu - m_K^2 - & \left[2a_1 n_p^s + (2a_2 + 4a_3)(n_p^s + n_n^s + n_{\Sigma^-}^s) \right. \\ & \left. + \left(\frac{5}{3}(a_1 + a_2) + 4a_3 \right) n_{\Lambda}^s \right] \frac{m_s}{2f^2} = 0. \end{aligned} \quad (83)$$

The first two terms in this equation are smaller than in the nucleons-only case, and this has to be compensated by the last term, which requires a higher density.

The presence of hyperons causes the condensate amplitude to increase rapidly with density (panel (d)); so rapidly, in fact, that large changes are induced in the scalar densities (panel(c)) and the Dirac effective masses of all the particles (panel(b)). The nucleon effective masses (see Eq. (50)) are given by

$$M_p^* = M - g_{\sigma n} \sigma + (2a_1 + 2a_2 + 4a_3)m_s \sin^2 \frac{1}{2}\theta,$$

$$M_n^* = M - g_{\sigma n} \sigma + (2a_2 + 4a_3) m_s \sin^2 \frac{1}{2} \theta , \quad (84)$$

and go to zero before the central stellar density is reached. This indicates the need to consider improvements, in particular, an exact evaluation of the zero-point energy with the non-linear Kaplan-Nelson Lagrangian, and this is under investigation.

Summarizing, we find that while the effect of non-zero temperature upon the onset of kaon condensation is small, both the presence of hyperons and neutrino trapping inhibit condensation, although the latter is a transient effect. It appears that in the MRHA, at least as formulated here, kaons play a much smaller role than suggested by non-relativistic treatments [6]. Mean field approximations give a larger effect, but we can not yet treat them satisfactorily when hyperons are present.

4.4.5 Sensitivity of kaon condensation to hyperon couplings

In the calculations above, we assumed that the couplings of the Σ and Ξ were equal to those of the Λ hyperon. Here, we relax this assumption and explore the sensitivity to unequal couplings of the different hyperons using the meson exchange formalism at zero temperature. Of the many possibilities, we pick three for study. These are listed in Table 15, in terms of the ratio to the nucleon couplings as defined earlier. For the Λ , we use the values discussed previously. For the Σ , we use two sets of values which gave satisfactory fits to the Σ^- atom data in the work of Mareš et al. [48]. This was based on a mean field description of nuclear matter using the nucleon couplings of Horowitz and Serot [49], who did not include non-linear terms ($U(\sigma) = 0$). The parameters for this model, termed ‘HS81’ here, are:

$$\frac{g_{\sigma N}}{m_\sigma} = 3.974 \text{ fm} , \quad \frac{g_{\omega N}}{m_\omega} = 3.477 \text{ fm} , \quad \text{and} \quad \frac{g_{\rho N}}{m_\rho} = 2.069 \text{ fm} . \quad (85)$$

Partly for consistency and partly because this model is often used as a baseline in the literature, we will adopt these parameters. (Qualitatively similar results are obtained for other values of the nucleon couplings, which yield more realistic values of the compression modulus.) Finally, we need the couplings of the Ξ . Since there is little information, we take the couplings to be equal to those of either the Λ or the Σ . Note that case 1 in this table is close to the set that we have been using in the previous discussion.

In Fig. 23, the particle fractions shown in the upper, center and lower panels refer, respectively, to hyperon coupling cases 1, 2 and 3 of Table 15. The upper panel is similar to results already discussed; note that kaons do not condense up to the maximum density displayed, $u = 4.5$. In discussing the other cases, we first mention the seeming paradox that increasing the coupling constants of a hyperon species delays its appearance to a higher density. The explanation [30, 31] is that the threshold equation receives contributions from the σ , ω and ρ mesons, the net result being positive due to the ω . Thus, if all the couplings

are scaled up, the positive contribution becomes larger, and the appearance of the particle is delayed to a higher density. With this in mind, consider the center panel of Fig. 23, which corresponds to case 2 of Table 15. The Σ couplings are larger than in case 1 (upper panel), so the Σ^- no longer appears, thus allowing the chemical potential μ to continue rising with density. This allows the Ξ^- to appear at $u = 2.2$, essentially substituting for the Σ^- . Of course, were we to reduce the Ξ couplings on the grounds that this hyperon contains two strange quarks, the Ξ^- would appear at an even lower density. Turning to the lower panel of this figure, we recall that this corresponds to case 3 of Table 15, for which both the Σ and Ξ couplings are increased. Neither of them now appear, and since the chemical potential, μ , continues to increase with density, it becomes favorable for kaons to condense at $u = 3.6$; the fraction Y_{K^-} , however, remains rather small.

Clearly, the lesson to be drawn from this is that the thresholds for the strange particles, hyperons and kaons, are sensitive to coupling constants that are poorly known. In matter where hyperons are allowed to be present, generally the effects of kaons are small. In fact, Schaffner and Mishustin [43] find that, with their choice of coupling constants, kaons do not condense. On the other hand, should it turn out that the coupling constants of the Σ and Ξ are larger than the ones adopted here as the standard choice, these hyperons might not be present at all, and consequently kaons would play a more important role. Thus, while strangeness plays a significant role in determining the constitution and physical properties of a neutron star, the detailed behavior cannot be tied down at the present time.

4.4.6 Metastability of neutron stars with kaon condensates

Fig. 24 shows the window of metastability in the baryonic mass, which, in the absence of mass accretion, is unchanged during the evolution of the star. Here the baryons are nucleons described in the mean field GM model without and with kaons, for which we use the chiral model with $a_3 m_s = -222$ MeV. When kaons are present, the range $M_B = 2.09 - 2.15 M_\odot$ can be supported by the initial EOS of lepton-rich matter, but not by the later EOS of lepton-poor matter (lines ending in dots). This range of metastability corresponds to gravitational masses of $M_G = 1.81 - 1.91 M_\odot$. In the absence of kaons, metastability does not occur, since the maximum mass decreases when the neutrinos leave (lines ending in stars). Note that the qualitative features here are similar to the case of matter with strangeness-bearing hyperons, see Fig. 15.

Fig. 25 shows the corresponding evolution of the maximum gravitational mass in matter with and without kaons in the deleptonization stage. As we have previously remarked, the initial state with $Y_{L_e} = 0.4$ corresponds to an electron-neutrino fraction of $Y_{\nu_e} \simeq 0.08$ and, of course, in the final state $Y_{\nu_e} = 0$. As in Fig. 16, the decrease (increase) in the maximum mass when kaons are present (absent) is most pronounced shortly before the neutrino fraction drops to zero.

4.5 Matter with quarks

We now examine another type of softening of high density matter by allowing for a hadron to quark phase transition in the interior of the star [8]. Glendenning [50] has suggested that a mixed phase of baryon and quark matter exists over a wide range of densities in the case of neutrino-free matter. Our primary interest here is in protoneutron stars and the effects of trapped neutrinos [8], which, conceivably, could lead to observable consequences. We focus on zero temperature, since, as we have seen, changes in the maximum mass due to neutrino trapping are larger than those due to finite temperature. The influence of complicated finite size structures due to Coulomb and surface effects [51] does not qualitatively affect our conclusions and will be taken up elsewhere.

We shall follow fairly closely the treatment of Glendenning [50]. Thus, for the pure phase in which the strongly interacting particles are baryons, we employ the mean field GM model, for which the formalism has been discussed in earlier sections. For the pure quark phase (in a uniform background of leptons), we use the bag model for which the pressure is

$$P_Q = -B + \frac{1}{3} \sum_{f=u,d,s} g_f \int_0^{k_{Ff}} \frac{d^3 k}{(2\pi)^3} \frac{k^2}{(k^2 + m_f^2)^{1/2}} + \frac{1}{3} \sum_{\ell} g_{\ell} \int_0^{k_{F\ell}} \frac{d^3 k}{(2\pi)^3} \frac{k^2}{(k^2 + m_{\ell}^2)^{1/2}}. \quad (86)$$

The first term accounts for the cavity pressure, and the second and third terms give the Fermi degeneracy pressures of quarks and leptons, respectively. The constant B has a simple interpretation as the thermodynamic potential of the vacuum, and will be regarded as a phenomenological parameter in the range $(100 - 250)$ MeV fm $^{-3}$. The lower limit here is dictated by the requirement that, at low density, hadronic matter is the preferred phase. For B much larger than the upper limit, a transition to matter with quarks never occurs. The degeneracy factor for quarks is $g_f = 2 \times 3$, accounting for the spin and color degrees of freedom. The chemical potential of free quarks in the cavity is $\mu_f = \sqrt{k_{Ff}^2 + m_f^2}$, where k_{Ff} is the Fermi momentum of quarks of flavor f . For numerical calculations, we take the u and d quarks as massless, and $m_s = 150$ MeV. The baryon density and the energy density are

$$\begin{aligned} n_Q &= \frac{1}{3} \sum_{f=u,d,s} n_f, & n_f &= \frac{k_{Ff}^3}{3\pi^2} \\ \varepsilon_Q &= -P_Q + \sum_f n_f \mu_f + \sum_{\ell} n_{\ell} \mu_{\ell}. \end{aligned} \quad (87)$$

The relevant weak decay processes in the pure quark phase are similar to Eq. (1), but with B_i replaced by q_f , where f runs over the quark flavors u , d , and s . In neutrino-free matter,

charge neutrality and chemical equilibrium under the weak processes imply

$$\sum_f q_f n_f + \sum_{\ell=e,\mu} q_\ell n_\ell = 0 \quad (88)$$

$$\mu_d = \mu_u + \mu_\ell = \mu_s. \quad (89)$$

When neutrinos are trapped, the new chemical equilibrium relation is obtained by the replacement $\mu_\ell \rightarrow \mu_\ell - \mu_{\nu_\ell}$ in Eq. (89).

In the mixed phase of hadrons and quarks, it is necessary to satisfy Gibbs' phase rules:

$$P_H = P_Q \quad \text{and} \quad \mu_n = \mu_u + 2\mu_d. \quad (90)$$

Further, following Glendenning [50], we require *global*, but not local, charge neutrality of bulk matter, for both separately conserved charges: baryon number and electric charge. Denoting by f the fraction of volume occupied by the hadronic phase, we have

$$f \sum_B q_B n_B + (1-f) \sum_{f=u,d,s} q_f n_f + \sum_{\ell=e,\mu} q_\ell n_\ell = 0 \quad (91)$$

$$n = f \sum_B n_B + (1-f) n_Q, \quad (92)$$

where q_B is the electric charge of each hadron. An important consequence of global neutrality is that baryonic and quark matter coexist for a much larger range of pressures than for the case of local charge neutrality [50]. The total energy density is $\varepsilon = f\varepsilon_H + (1-f)\varepsilon_Q$.

4.5.1 Metastability of neutron stars with quarks

Fig. 26 shows a comparison of the compositions of neutrino-free matter (top panel) and neutrino-trapped matter (bottom panel). In the case of neutrino-free matter, quarks make their appearance at around $4n_0$ for $B = 200$ MeV fm $^{-3}$. After this, the neutral and negative particle abundances begin to fall, since quarks furnish both negative charge and baryon number. The bottom panel of Fig. 26 shows the influence of trapped neutrinos (with $Y_{Le} = 0.4$) on the relative fractions. The primary role of trapped neutrinos is to increase the proton and electron abundances, which strongly influences the threshold for the appearance of hyperons. The Λ and the Σ 's now appear at densities higher than those found in the absence of neutrinos. In addition, the transition to a mixed phase with quarks is delayed to about $10n_0$. Qualitatively similar trends are observed for other values of the bag constant B .

In Fig. 27, we show the phase boundaries as a function of the bag pressure B . The onset of the phase transition is at density $n_1 = u_1 n_0$, and a pure quark phase begins at density $n_2 = u_2 n_0$. Also shown are the central densities $n_c = u_c n_0$ of the maximum mass

stars. Trapping shifts the onset of the phase transition to higher baryon densities and also reduces the extent of the mixed phase in comparison to the case of neutrino-free matter. The existence of a mixed phase inside the star depends on whether or not hyperons are present. In the absence of hyperons (top panel), a mixed phase is present for the entire range of bag constants. When hyperons are present (bottom panel), the mixed phase is present only when B is sufficiently low ($B \leq 165 \text{ MeV fm}^{-3}$) and occurs over a smaller range in density than that found in the absence of hyperons. The abrupt change in the onset of the transition around $B = 140 \text{ MeV fm}^{-3}$ is caused by the appearance of hyperons prior to that of quarks.

The dashed lines in these figures correspond to the case in which neutrinos have left the star. Whether or not hyperons are present, the mixed phase is now present over a wide range of density inside the star. Note also that, since the central density of the star $u_c < u_2$ for all cases considered, the presence of a pure quark phase is precluded. Finally, note that quarks are more likely to appear the fewer the neutrinos remaining in the star.

Table 16 shows the maximum masses of stars as a function of the composition of the matter. With only nucleons and leptons (last row), neutrino trapping generally *reduces* the maximum mass from the case of neutrino-free matter. This is caused by the smaller pressure support of lepton-rich matter, in which the gain in the negative symmetry pressure exceeds the increase in leptonic pressure. However, the introduction of quarks, which soften the EOS, causes the maximum mass for the trapped case to be *larger* than that for neutrino-free matter. This reversal in behavior is due to the fact that the first appearance of quarks occurs at a higher density when neutrinos are trapped. When hyperons are present, the maximum mass remains larger for the trapped case. This is also due to the appearance of hyperons at a higher density when neutrinos are trapped. It is an example of the general result that when matter contains non-leptonic negative charges, the maximum mass of the neutrino-trapped star is larger than that of the neutrino-free star. This result has important ramifications for the evolution of proto-neutron stars and for the formation of black holes, as we discuss in Sec. 6.

4.6 Global energetics

While the softening of the EOS due to the presence of negatively-charged particles has a large effect upon the maximum mass, it has surprisingly little effect upon the binding energy versus mass relationship for neutron stars. The binding energy is the difference between baryonic and gravitational masses of the final neutron star configuration. It is an important observational parameter, because at least 99% of it appears as radiated neutrino energy. In Fig. 28, we display the binding energy as a function of the baryonic mass M_B for stars with and without the presence of strangeness-bearing components, such as kaons

or hyperons. The various curves refer to different EOSs and terminate at the maximum mass supported by their respective EOS.

A few striking results are evident from this figure.

1. The largest binding energy occurs for the EOS that supports the largest maximum mass.
2. For each EOS, the binding energy displays a nearly quadratic behavior up to the maximum mass.
3. There exists a rather narrow band of possible binding energies for a given mass, implying the following universal relationship for the binding energy as a function of mass:

$$B.E. = (M_B - M_G)c^2 \cong (0.065 \pm 0.01) \left(\frac{M_B}{M_\odot} \right)^2 M_\odot, \quad (93)$$

where the numerical coefficient represents an update of the value quoted earlier by Lattimer and Yahil [52]. The universality is not altered by the presence of significant softening in the high density EOS due to the appearance of quarks, kaons, or hyperons.

4. Only near the terminations at the maximum masses do the binding energies slightly deviate from the lower envelope of the curves. This effect is slightly more pronounced for softer equations of state.

We have found that the lower envelope of the binding energy–mass relation is equivalent to that found for the stiffest plausible equation of state, namely one that is limited by causality. Denoting by n_t a transition density above which the EOS is assumed to be causal, the EOS above n_t is given by [53]:

$$\begin{aligned} P &= \frac{1}{2} \left[P_t - \epsilon_t + (P_t + \epsilon_t) \left(\frac{n}{n_t} \right)^2 \right]; \\ \epsilon &= \epsilon_t + P - P_t, \end{aligned} \quad (94)$$

where the quantities P_t and ϵ_t are the pressure and energy density at n_t and thus depend both upon it and upon the equation of state employed. However, if n_t is in the range $n_0 - 2n_0$, both P_t and ϵ_t are somewhat insensitive to the EOS, and the binding energy is relatively insensitive to these quantities. Fig. 28 includes the binding energy as a function of baryon mass for an EOS which is causal above $n_t = 0.3 \text{ fm}^{-3}$ and matched to the GM equation of state below n_t . Varying the values of n_t and the EOS below n_t merely alters the

termination point (maximum mass) of this curve, without otherwise noticeably changing it.

We note that an analytic representation of the binding energy–mass relation can be determined using the techniques developed by Nauenberg and Chapline [54], who assumed that the pressure and energy density are constant within the neutron star as an alternative to an explicit integration of the Tolman-Oppenheimer-Volkov equation [14]. Defining the parametric variable χ by

$$\sin^2 \chi = \frac{2GM}{Rc^2}, \quad (95)$$

the mass is

$$M = \sqrt{\frac{3c^6(1-\xi)}{32\pi G^3(\epsilon_t - P_t)}} \sin^3 \chi; \quad \xi = \frac{6 \cos \chi}{9 \cos \chi - Q} - 1, \quad (96)$$

where $Q = 2 \sin^3 \chi / (\chi - \sin \chi \cos \chi)$. The binding energy can then be written as

$$B.E. = M \left[\frac{3n_t m_B c^2}{\sqrt{\epsilon_t^2 - P_t^2}} \frac{\sqrt{1-\xi^2}}{Q} - 1 \right]. \quad (97)$$

Utilizing $Q \cong 3(1 - 3\chi^2/10 + \dots)$ and $\xi \cong \chi^2/10 + \dots$, as appropriate for low mass stars, one recovers the Newtonian result that $B.E. = (3/5)GM^2/Rc^2 + \dots$, which displays the quadratic dependence on the mass of the star.

Since nearly all of the binding energy is released in the form of neutrinos, it appears that an accurate measurement of the total radiated neutrino energy will lead to a good estimate of the remnant mass. However, as the results of Fig. 28 and Eq. (93) show, it will not be possible to distinguish the various equations of state from the total binding energy alone.

5 Evolution timescales

We have seen that the structure of a neutron star is strongly influenced by the presence of trapped neutrinos and, to a lesser extent, by the non-zero entropy/baryon. The evolution of the star will be governed by the timescale for release of the trapped neutrinos, referred to as deleptonization, and the timescale for thermal cooling, which reduces the entropy to a small value. We discussed these timescales in Sec. 1 on the basis of detailed numerical calculations of the evolution dynamics [1]. Since the timescales are of some significance, we will show in this section how these can be estimated from analytical considerations and illustrate the dependence on the equation of state and the opacities.

The equations that describe the physical state and evolution of the nascent neutron star are simply those of standard stellar structure theory, modified for the effects of general relativity and augmented to include lepton and neutrino transport. (Photon transport is completely suppressed at the high densities of the core.) All the known neutrino species and their antiparticles carry energy. Most of the time, in most of the star, the neutrinos are in thermal equilibrium with the matter and have Fermi distributions. This is true because neutrino processes, such as the nucleon Urca, pair production, and inverse nucleon brehmsstrahlung, are sufficiently rapid. The calculations of Maxwell [55], for example, illustrate that the timescale for ν_e (ν_μ) equilibration will be less than 1 s, at nuclear densities, for temperatures $T \geq 2(3)$ MeV. This is an overestimate for the equilibration time for ν_e , since Maxwell did not consider the direct Urca process. If that process is included, the timescale decreases by an order of magnitude.

It is a good approximation to lump together ν_μ and ν_τ transport together into “ μ ” transport, and thereby assume them to have zero chemical potential and equal opacities. Further simplification can be made if one assumes the electron neutrino and antineutrino opacities are also equal. Then we need only define one e -type chemical potential, denoted by μ_ν . This is exact deep in the opaque interior, but breaks down in the transparent regime above the neutrinosphere. It can be joined smoothly to a free-streaming approximation here, however. The relevant equations, in spherical symmetry, have been given in Ref. [1], to which we refer the reader for more elaborate discussion. They are

$$\frac{dP}{dr} = -\frac{G(M + 4\pi r^3 P)(\rho + P/c^2)}{r(r - 2GM/c^2)} \quad (98)$$

$$\frac{dM}{dr} = 4\pi r^2 \rho \quad (99)$$

$$\frac{dN}{dr} = \frac{4\pi r^2 n}{\sqrt{1 - 2GM/rc^2}} \quad (100)$$

$$\frac{dY_\nu}{d\tau} = -e^{-\phi} \frac{\partial(4\pi r^2 F_\nu e^\phi)}{\partial N} + S_\nu \quad (101)$$

$$\frac{dY_e}{d\tau} = -S_\nu \quad (102)$$

$$\frac{dU}{d\tau} = -P \frac{d(1/n)}{d\tau} - e^{-2\phi} \frac{\partial L_\nu e^{2\phi}}{\partial N}. \quad (103)$$

Here Eq. (98) is the general relativistic equation for hydrostatic equilibrium, in which $M(r)$ is the enclosed gravitational mass. The enclosed baryon mass, $N(r)$, obeys Eq. (100). Eqs. (101) and (102) give the rate of change of the electron neutrino and electron concentrations, with F_ν the number flux of electron neutrinos and S_ν the electron neutrino source term. Finally, Eq. (103) gives the rate of change of U , the internal energy per baryon, where L_ν

is the total neutrino luminosity (including all species). The term $e^\phi = \sqrt{-g_{00}}$ relates time at infinity τ with the coordinate time t , and one can show [1] that $d\phi/dP = -(P + \rho c^2)^{-1}$.

In the diffusion approximation, fluxes are driven by density gradients. In our context, this translates into expressions of the form

$$F_\nu = - \int_0^\infty \frac{c\lambda_\nu}{3} \frac{\partial n_\nu(E_\nu)}{\partial r} dE_\nu; \quad (104)$$

$$L_\nu = - \int_0^\infty 4\pi r^2 \sum_i \frac{c\lambda_E^i}{3} \frac{\partial \epsilon_i(E_\nu)}{\partial r} dE_\nu, \quad (105)$$

where the sum is over neutrino species. The λ_ν and λ_E^i 's are mean free paths for number and energy transport, respectively, and are functions of neutrino energy E_ν . Also, $n_\nu(E_\nu)$ is the number of electron neutrinos with energy E_ν , and $\epsilon_i(E_\nu)$ is the energy density of species $i = e, \mu$. The general relativistic corrections have been dropped for clarity, although they are straightforward to incorporate.

We can combine Eqs. (101) and (102) to obtain the rate of change of the total lepton number, and Eq. (103) and the first law of thermodynamics to obtain the rate of change of the entropy:

$$n \frac{dY_L}{dt} = n \left(\frac{dY_e}{dt} + \frac{dY_\nu}{dt} \right) = - \frac{1}{r^2} \frac{\partial}{\partial r} r^2 F_\nu \quad (106)$$

$$nT \frac{ds}{dt} = - \frac{1}{4\pi r^2} \frac{\partial L_\nu}{\partial r} - n \sum_{i=n,p,e,\nu} \mu_i \frac{dY_i}{dt}. \quad (107)$$

To proceed, we have to understand the energy dependence of the opacities. There are two main sources of opacity:

1. Neutrino-nucleon absorption. This affects ν_e and $\bar{\nu}_e$ only, except at very high densities if muons are present, which is not possible until relatively late in the cooling. The absorption mean free path, assuming nondegenerate nucleons, is

$$\lambda_{abs} = \lambda_{abs}^o \left(\frac{E_{\nu o}}{E_\nu} \right)^2 \text{ cm}, \quad (108)$$

where λ_{abs}^o is the fiducial absorption mean free path at the fiducial ν_e energy $E_{\nu o} \simeq 260$ MeV, the typical ν_e chemical potential at the beginning of deleptonization. From Ref. [56], we find

$$\lambda_{abs}^o = \frac{4}{n_n \sigma_o (1 + 3g_A^2)} \left(\frac{m_e c^2}{E_{\nu o}} \right)^2, \quad (109)$$

where n_n is the neutron number density, $\sigma_o = 1.76 \times 10^{-44} \text{ cm}^2$, and $g_A \simeq 1.257$. Using $n_n = (8/3)n_0$, appropriate for $Y_e = 1/3$ and a total baryon density $n = 4n_0$ at the beginning of deleptonization, we find $\lambda_{abs}^o \simeq 0.36 \text{ cm}$. Since nucleons will, in fact, be at least partially degenerate, and because of fermi liquid effects, the true absorption mean free path will be about 3–10 times larger than this value.

2. Neutrino-nucleon scattering. This elastic scattering affects all ν -types. For nondegenerate nucleons [56],

$$\lambda_{es}^o = \frac{4}{n\sigma_o} \left(\frac{m_e c^2}{E_{\nu o}} \right)^2, \quad (110)$$

$$\lambda_{\mu s}^o = \frac{4}{n\sigma_o} \left(\frac{m_e c^2}{E_{\nu_{\mu o}}} \right)^2, \quad (111)$$

for ν_e and ν_μ , respectively. For our reference density, $4n_0$, we obtain $\lambda_{es}^o \simeq 1.37 \text{ cm}$ for $E_{\nu o} = 260 \text{ MeV}$, and $\lambda_{\mu s}^o \simeq 1.75 \text{ cm}$ for $E_{\nu_{\mu o}} = 230 \text{ MeV}$; the latter is the appropriate value for the mean ν_μ energy, with $s = 2$, at the beginning of the cooling era. Corrections to the scattering mean free paths for degeneracy and interactions should be similar to those for absorption.

Thus, during deleptonization, $\lambda_{abs}^o < \lambda_s^o$, and ν_e absorption dominates both energy and lepton number transport. However, during thermal cooling, energy transport is effected mostly by $\mu-$ and $\tau-$ neutrinos, since ν_e 's are more tightly coupled to the matter.

The opacities imply three kinds of fluxes:

1. a number flux F_ν of ν_e 's, dominated by absorption. Utilizing Eq. (108), we have

$$-F_\nu = \int_0^\infty \frac{c\lambda_{abs}}{3} \frac{\partial n_\nu(E_\nu)}{\partial r} dE_\nu = \frac{c\lambda_{abs}^o E_{\nu o}^2}{6\pi^2(\hbar c)^3} \frac{\partial \mu_\nu}{\partial r} \equiv a \frac{\partial \mu_\nu}{\partial r}. \quad (112)$$

2. an energy flux L_ν^e of ν_e 's, also dominated by absorption:

$$-L_\nu^e = \int_0^\infty 4\pi r^2 \frac{c\lambda_{abs}}{3} \frac{\partial \epsilon_\nu(E_\nu)}{\partial r} dE_\nu = 4\pi r^2 a \frac{\partial}{\partial r} \left(\frac{\pi^2 T^2}{6} + \frac{\mu_\nu^2}{2} \right). \quad (113)$$

3. an energy flux L_ν^μ of ν_μ 's and ν_τ 's, dominated by scattering:

$$\begin{aligned} -L_\nu^\mu &= 2 \int_0^\infty 4\pi r^2 \frac{c\lambda_{\mu s}^o}{3} \frac{\partial \epsilon_{\nu_\mu}(E_\nu)}{\partial r} dE_\nu &= 4\pi r^2 \frac{c\lambda_{\mu s}^o (E_{\nu_{\mu o}})^2}{3\pi^2(\hbar c)^3} \frac{\partial}{\partial r} \left(\frac{\pi^2 T^2}{6} \right) \\ &\equiv 4\pi r^2 b \frac{\partial}{\partial r} \left(\frac{\pi^2 T^2}{6} \right), \end{aligned} \quad (114)$$

where we used the fact that ν_μ 's and ν_τ 's have zero chemical potential.

5.1 Deleptonization Era

We can now appreciate the separate stages of deleptonization and cooling. Deleptonization is dominated by number transport, and the controlling equation is

$$n \frac{dY_L}{dt} = n \frac{\partial Y_L}{\partial Y_\nu} \frac{dY_\nu}{dt} = \frac{a}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \mu_\nu}{\partial r} \right). \quad (115)$$

During deleptonization, the electron neutrinos are degenerate, and to lowest order $nY_\nu \simeq (\mu_\nu/\hbar c)^3/6\pi^2$. The neutrinos and electrons are in beta equilibrium, and it can be shown that $\partial Y_L/\partial Y_\nu = (\partial Y_L/\partial Y_\nu)_o (E_{\nu o}/\mu_\nu)$, where $(\partial Y_L/\partial Y_\nu)_o \simeq 3$ for $Y_{\nu o} \simeq 0.06$. Thus,

$$3E_{\nu o} \left(\frac{\partial Y_L}{\partial Y_\nu} \right)_o \mu_\nu \frac{\partial \mu_\nu}{\partial t} = \frac{a}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \mu_\nu}{\partial r} \right). \quad (116)$$

We now seek separable solutions of the form $\mu_\nu = E_{\nu o} \phi(t) \psi(r)$. We obtain

$$3 \left(\frac{\partial Y_L}{\partial Y_\nu} \right)_o \frac{E_{\nu o}^2}{a} \frac{\partial \phi}{\partial t} = \frac{1}{r^2 \psi^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) = -\alpha, \quad (117)$$

where α is a separation constant. One sees that

$$\phi = 1 - t/\tau_d ; \quad \tau_d = \frac{3}{c \lambda_{abs}^o \alpha} \left(\frac{\partial Y_L}{\partial Y_\nu} \right)_o, \quad (118)$$

where τ_d is the diffusion time. Note that the decay of the neutrino chemical potential is approximately linear with time. This result is borne out in more detailed numerical calculations. The radial equation is

$$-\alpha R^2 \psi^2 = \frac{1}{x^2} \frac{\partial}{\partial x} \left(x^2 \frac{\partial \psi}{\partial x} \right), \quad (119)$$

which is just that of the Lane-Emden polytrope [57] of index 2. In the above, R is the radius of the star. The solution of this equation satisfies $\alpha R^2 \simeq 19$; and, therefore,

$$\tau_d \simeq \frac{9R^2}{19c\lambda_{abs}^o} \simeq 44.3 \left(\frac{R}{10 \text{ km}} \right)^2 \text{ s.} \quad (120)$$

Recalling that degeneracy and fermi liquid corrections will increase the mean free path by a factor of 3–10, a deleptonization time of 5–15 s is indicated. This is the correct magnitude for the deleptonization time, and shows clearly how it depends upon the equation of state through the radius of the star, R , and upon the opacity.

Because of the positive temperature gradient and the chemical potential gradient, the deleptonization is accompanied by heating in the core. The entropy/baryon rises to the value of about 2 before it decreases during the cooling era. The onset of cooling does not begin until deleptonization is complete. Rewriting the last term of Eq. (107) as

$$-n \sum_{n,p,e,\nu} \mu_i \frac{dY_i}{dt} = n(\mu_n - \mu_p - \mu_e + \mu_\nu) \frac{dY_e}{dt} - \mu_\nu \frac{dY_L}{dt}, \quad (121)$$

we can combine Eqs. (103) and (104) to find

$$nT \frac{ds}{dt} = \frac{a+b}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial(\pi^2 T^2/6)}{\partial r} \right) + a \left(\frac{\partial \mu_\nu}{\partial r} \right)^2. \quad (122)$$

The terms proportional to a are due to electron neutrinos, and the term proportional to b is due to the other neutrinos. There is heating or cooling depending on the direction of the temperature gradient, but the chemical potential gradients always lead to heating. When $\mu_\nu \gg T$, the $(\partial \mu_\nu / \partial r)^2$ term dominates, and we have heating. When $\mu_\nu \simeq 0$, and the temperature decreases with radius, cooling occurs.

5.2 Thermal Cooling Era

We now turn to the thermal cooling of the protoneutron star, which continues beyond the deleptonization era. While the initial entropy per baryon s in the star's interior is about 1, after the deleptonization heating is finished the entropy reaches the value of about 2. The entropy is dominated by baryons for temperatures less than about 100 MeV. Thus, we may write [27]

$$s \approx 2a_{\ell d}T ; \quad a_{\ell d} = \frac{1}{15} \frac{m^*}{m} \left(\frac{n_0}{n} \right)^{2/3} \text{ MeV}^{-1}, \quad (123)$$

where m^* is the effective nucleon mass. Note that the maximum value of the central temperature is

$$T_{max} = \frac{s_{max}}{2a_{\ell d}} \simeq 37.8 s_{max} \left(\frac{m}{2m^*} \right) \left(\frac{n}{4n_0} \right)^{2/3} \text{ MeV}. \quad (124)$$

We henceforth neglect the density dependence of m^* and use $m^* \simeq 0.5m$. Notice that the estimate of T_{max} in Eq. (124) agrees quite well with our previously tabulated results.

The cooling is dominated by the μ - and τ -neutrinos, since $b > a$. With these simplifications, Eq. (122) becomes

$$\frac{6a_{\ell d}n}{\pi^2(a+b)} \frac{\partial T^2}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T^2}{\partial r} \right). \quad (125)$$

Once again, we may separate the resulting equation in terms of the time and radial dependence of the temperature. Writing $T = T_{max}\psi(r)\phi(t)$, we find

$$\frac{12a_{ld}n}{\pi^2(a+b)}\frac{1}{\phi}\frac{\partial\phi}{\partial t} = \frac{1}{\psi^2r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi^2}{\partial r}\right) = -\alpha, \quad (126)$$

where α is a separation constant. The solution of the radial equation is that of an $n = 1$ Lane-Emden polytrope, for which the eigenvalue is $\alpha = \pi^2/R^2$. The temporal equation has the solution

$$\phi = \exp(-(t - \tau_d)/\tau_c) \quad (127)$$

for $t > \tau_d$, where

$$\tau_c = \frac{12a_{ld}nR^2}{\pi^4(a+b)} = \frac{3s_{max}(E_{\nu o}/T_{max})R^2}{Y_{\nu o}\pi^4c(\lambda_{abs}^o/2 + \lambda_{\mu s}^o(E_{\nu \mu o}/E_{\nu o})^2)} . \quad (128)$$

Note that

$$\frac{\tau_c}{\tau_d} = \frac{19s_{max}}{3Y_{\nu o}\pi^4}\frac{E_{\nu o}}{T_{max}}\frac{1}{1/2 + (\lambda_{\mu s}^o/\lambda_{abs}^o)(E_{\nu \mu o}/E_{\nu o})^2} \simeq 1.7 . \quad (129)$$

This result is independent of our assumptions regarding s_{max} or T_{max} . This is also of the right magnitude to match the numerical calculations, which indicate that $\tau_c = (1 - 2)\tau_d$. Thus, in spite of the fact that the mean free paths that dominate cooling are larger than those that dominate deleptonization, the large ratio of the matter's heat capacity to that of the neutrinos forces the cooling time to be longer than the deleptonization time.

At late times, however, the decay of the central entropy or temperature is roughly linear with time [1]. This feature can be seen to be a result of the increasing degeneracy of the star. For degenerate nucleons, the mean free paths have an E_{ν}^{-3} dependence. Ignoring the absorption contributions, this energy dependence leads to a linear time decay, with a time constant

$$\tau'_c = \frac{s_{max}R^2}{76(\ln 2)Y_{\nu o}c\lambda'_s} \left(\frac{E_{\nu o}}{E_{\nu \mu o}}\right)^3 , \quad (130)$$

where the fiducial scattering mean free path for degenerate nucleons is [56]

$$\lambda'_s = \lambda_{\mu s}^o \frac{5}{1 + 4g_A^2} \left(\frac{p_{F_n}}{E_{\nu \mu o}}\right) \simeq 1.6\lambda_{\mu s}^o . \quad (131)$$

For the values we have been assuming, one finds that $\tau'_c \simeq 11$ s. Of course, since this result is applicable to the later stages of cooling, the use of a smaller value of $E_{\nu \mu o}$ may

be appropriate, which will lead to an increase in this timescale estimate. In any case, one expects the experimental behavior of Eq. (129) to alter to a linear decay after an e -folding time.

It is finally interesting to note the overall sensitivity to the equation of state. For a fixed mass star, employing $R \propto n_c^{-1/3}$, where n_c is the central density, we determine that $\tau_d \propto n_c^{1/3}$ and $\tau_c \propto n_c^0$ so that for nucleons-only matter, the central density affects the timescales rather weakly. However, larger changes in the opacities and radii are triggered by the appearance of negatively charged strongly interacting particles in any form [58], suggesting that their onset may significantly affect these timescales.

6 Implications

Our main thrust in this work has been to elucidate how the structure of a proto-neutron star depends on its composition, which is chiefly determined through the nature of the strong interactions at high baryon density. During its early evolution, a neutron star with an entropy per baryon of order unity contains neutrinos that are trapped in matter on dynamical timescales. After a time of a few tens of seconds, the star achieves its cold, catalyzed structure with essentially zero temperature and no trapped neutrinos. The influence of finite temperature on the star's structure is dominated by the behavior of the baryonic thermal pressures, which are governed by the behavior of the baryonic effective masses. Baryonic thermal pressures are proportional to their Landau effective masses, so that nuclear models that lead to extremely small effective masses at high density, such as Skyrme-type interactions, will generally show substantially larger effects at finite temperature than other models. We have shown, however, that the gross reduction of the effective mass in the Skyrme case leads to acausal sound speeds in dense matter and should be discounted. Thus, finite temperature effects upon the maximum neutron star mass are naturally limited.

In general, however, changes in the maximum mass due to neutrino trapping are larger than those due to finite temperatures. These changes depend sensitively on the composition of matter, in particular, on the question of whether or not a new component that substantially softens matter can appear in the cold, catalyzed star at high density. The new components that have been discussed to date include hyperons, a pion or kaon condensate, and a transition to quark matter. All these components involve negatively charged non-leptonic matter; hence, they appear at lower density in the cold, catalyzed star than in the hot, neutrino-trapped star. Since thermal pressure is always positive, the cold, catalyzed star always has a higher density than a hot, neutrino-trapped star. Consequently, a cold, catalyzed star contains the softening component in a larger proportion of the star's mass than the hot, neutrino-trapped star, and this leads to a *smaller* maximum mass.

This behavior is opposite to that found for equations of state containing only nucleons and leptons and no additional softening component. In this case, neutrino trapping generally reduces the maximum mass from the value found in neutrino free matter; although neutrino-trapped matter contains more leptons and more leptonic pressure, it also contains more protons and, therefore, less baryonic symmetry pressure. While finite entropy provides additional pressure support, the amount of the increase over the zero temperature case is generally small, especially if realistic forces are employed.

It must be emphasized that the maximum mass of the cold catalyzed star still remains uncertain, due to the uncertainty in strong interactions at high density. At present, all nuclear models can only be effectively constrained at nuclear density and by the condition of causality at high density. The resulting uncertainty is evident from the range of possible maximum masses predicted by the different models considered in this work. Despite this uncertainty, our findings concerning the effects of finite entropy and neutrino-trapping offer intriguing possibilities for distinguishing between the different physical states of matter. These possibilities include both black hole formation in supernovae and the signature of neutrinos to be expected from supernovae, as we now discuss.

6.1 Black hole formation

The gravitational collapse of the core of a massive star produces a lepton-rich, neutrino trapped, proto-neutron star and an expanding shock wave. Energy losses from dissociation of heavy nuclei and neutrinos weaken the shock, preventing a “prompt” explosion. Within a few milliseconds, the shock wave stagnates into an accretion shock at a distance of 100-200 km from the proto-neutron star. Gandhi and Burrows [59] have demonstrated that the neutrino luminosity of the stellar remnant is able to quasi-statically support the shock against the ram pressure of infalling matter, which accretes onto the neutron star. Recent successful models of gravitational collapse supernovae [60, 61] invoke delayed neutrino heating, augmented by convective motions, to power the supernova explosion.

The explosion appears to occur within $\frac{1}{2}$ to a few seconds after the core bounce; and, once expansion occurs, accretion onto the proto-neutron star is diminished. Therefore, the star can be expected to accumulate nearly all of its baryon number within a few seconds of core bounce.

As we have seen, a nucleons-only star has a maximum mass that *grows* as neutrinos leak out of it. It thus appears unlikely that such a star could form a black hole during the longer-term deleptonization era. In this case, a black hole could only be produced immediately after bounce or during the short-term accretion stage, when the shock is stagnant.

However, this is not the case for matter with non-leptonic, negatively charged softening components. For matter containing hyperons, kaons, or quarks, the maximum mass of the neutrino-trapped star is larger than that of the neutrino-free star. Should the maximum

baryon mass of the cold neutrino-free star be close to $1.5M_{\odot}$, black hole formation could occur as the neutrinos diffuse out of the protoneutron star, i.e., during the first 10 seconds following bounce. Burrows [5] has demonstrated that black hole formation should be accompanied by a dramatic cessation of the neutrino signal, since the event horizon invariably forms outside the neutrinosphere. Such behavior would be relatively easy to observe from a galactic supernova and would suggest that the equation of state produced a metastable protoneutron star.

In order to highlight some of the observable consequences for different compositions of high density matter, the gravitational and baryonic masses, and their differences, both for cold, catalyzed matter and for hot, neutrino-rich matter, are shown in Fig. 29. Two generic compositional cases are displayed: nucleons-only matter ($npe^{-}\mu^{-}$), and matter with hyperons ($npHe^{-}\mu^{-}$). Matter with condensed kaons and with a quark-hadron transition has a similar behavior, and is discussed in more detail in Refs. [6, 8].

In all cases, the gravitational mass for the neutrino-rich cases are greater than for the untrapped case for the same baryon mass, reflecting the binding energy released during this stage of neutron star formation. This is usually less than half the total binding energy released, the remainder being emitted during the prior stage. The prior stage is the period between core bounce and the production of the approximately adiabatic ($s \approx 2$), lepton-rich star (shown by the upper solid curves in Fig. 29), and lasts about 1-3 seconds. The binding energy emitted during the prior stage is difficult to show in Fig. 29, because during this period the neutron star is rapidly accreting mass (i.e., M_B is increasing). The accretion should drop substantially, and the value of M_B should approach a limiting value after the first few seconds. We note that the binding energy emitted in the deleptonization and cooling stage appears to be insensitive to the composition of dense matter, just as the total binding energy was found to be a universal function of mass. Given its increase with stellar mass, this means that the total energy released is apparently not a good discriminant of composition.

However, Fig. 29 clearly shows the consequence of different compositions for black hole formation. Black hole formation can be observed as an abrupt cessation of neutrino signal, since the event horizon forms outside of the star's neutrinosphere. For the nucleons-only case, black hole formation is unlikely to occur during the deleptonization and cooling stage, the one marked by the transition between the upper solid and the lower dashed curves, since M_B is approximately constant (or only slightly increasing). If a black hole were to form from a star with this composition, it is much more likely to form during the post-bounce accretion stage. This is not true for the other compositional cases. Here, the neutrino-trapped matter is always capable of supporting more mass than the cold, catalyzed matter. If the baryon mass of the proto-neutron star is near the maximum mass of the cold, catalyzed neutron star, as it is, for example, for a $1.5 M_{\odot}$ star in both the $npHe^{-}\mu^{-}$ and the $npHQe^{-}\mu^{-}$ cases, then there exists a range of $0.1 - 0.2 M_{\odot}$ above this mass in which a hot neutrino-

trapped star can be stabilized. Thus, with this composition, a black hole could form during either stage of neutron star formation; although, given the relatively small value expected for the mass of the imploding pre-bounce core, it seems more likely that a black hole would form during the later stage.

A consequence of the potentially long delay of 10–15 seconds between core bounce and black hole formation is that black hole formation and total binding energy release are not necessarily correlated. The softening in the equation of state marked by the appearance of negatively charged hadrons is accompanied by relatively little further binding energy release. Thus, the large energy release inferred by the neutrino detections from SN 1987A did not imply that the equation of state could not yet change due to the appearance of any exotic matter.

Black hole formation, by cutting off the neutrino luminosity from the protoneutron star, would short-circuit the supernova mechanism if an explosion had not already occurred. This appears to be more likely for the nucleons-only stars or for stars with relatively large initial core masses, i.e., very massive stars with $M \geq 20 - 30 M_{\odot}$ [62, 63]. Note also that these scenarios have different implications for nucleosynthesis, since prompt black hole formation and a successful supernova explosion, in which newly synthesized nuclei are ejected, may be incompatible.

Based on the calculations of Prakash and Lattimer, Brown and Bethe [11] proposed that a window exists in which neutron stars collapse to black holes during deleptonization if hyperons, kaons or quarks are present in neutron stars. This proposal was meant to explain the apparent non-existence of a neutron star in the remnant of SN1987A, although a neutron star may have temporarily existed some 10–15 s, during which neutrino emission was observed. The window naturally exists if negatively charged hadronic matter appears after deleptonization. The particular case of SN1987A will be discussed below.

6.2 Neutrino signals from supernovae

The composition of a neutron star will also influence the details of the star’s neutrino emission. We focus on two possible diagnostics – the total radiated energy and the relative numbers of emitted neutrinos of different types.

In Fig. 30, we display the electron concentration, Y_e , for various dense matter compositions. The reference straight line is the total electron-lepton concentration of the initial proto-neutron star, which we have assumed to be approximately $Y_{Le} = 0.4$. The difference in any of the other curves from the reference line shows, as a function of density, the total net electron-neutrino concentration (specifically, the difference of the ν_e and $\bar{\nu}_e$ concentrations) that eventually leaks out of dense matter. The integral of the difference over the stellar density profile gives the net number difference for the entire star. This difference is always positive, although the actual value is sensitive to composition.

A rough measure of the relative fluxes of escaping neutrinos can be found as follows. Although, in the stellar core, there are essentially only ν_e 's because $\mu_e/T \gg 1$, $\mu_e \gg 1$ and $\mu_\mu = \mu_\tau \simeq 0$, the emerging neutrino signature consists of nearly equal proportions of all six types of neutrinos. This may be understood as follows. Diffusion degrades the high energies ($\mu_{\nu_e} \approx 200$ MeV) of the core ν_e 's; therefore, the emerging neutrinos have average energies in the range 10-20 MeV. Pair production in the hot matter in the outer mantle of the proto-neutron star generates several pairs of all three neutrino flavors per core ν_e . Using 10 MeV for the emergent energy, we find about 3 pairs emerge per core ν_e emitted. This shows that the total number of escaping neutrinos has a slight excess of ν_e 's; for this example, the ratio of $\nu_e : \nu_x$, where x refers to any of the other neutrino species, is about 4:3.

The actual situation is more complicated because, in general, different neutrino species are emitted with different energies. Also, the total flux of neutrinos should be sensitive to the available binding energy change. Nevertheless, the basic trends are clear: First, the smaller the value of Y_e in the cold, catalyzed neutron star, the larger the excess of ν_e 's that will be emitted. Second, the larger the binding energy change during deleptonization, the *smaller* the relative excess of ν_e 's will be. This is because the energies of the escaping neutrinos are rather insensitive to the structure of the proto-neutron star, including details of the equation of state of high density matter. The energies of the escaping neutrinos are determined instead by the properties of the outer mantle of the star. Thus, higher binding energy release translates directly to larger total numbers of escaping neutrinos.

However, referring to Fig. 29, one sees that the change in binding energies during deleptonization and cooling is relatively insensitive to the dense matter composition, although it does increase with the final neutron star mass. Therefore, the net excess of ν_e 's from a proto-neutron star during deleptonization and cooling seems to be a probe of the final value of Y_e in dense matter, a quantity that is quite sensitive to dense matter composition. It remains to be seen if differences in the final value of Y_e are large enough to be observable in the neutrino signal from a galactic supernova.

6.3 Supernova SN1987A

The case of SN1987A is interesting in light of the potential metastability of forming neutron stars. On the one hand, on February 23 of 1987 neutrinos were observed from the explosion of supernova SN1987A, indicating that a neutron star, not a black hole, was initially present. On the other hand, the ever-decreasing optical luminosity of the remnant of SN1987A suggests two arguments [64, 65] against the presence of a neutron star.

First, accretion at the Eddington limit with the usual Thomson electron scattering opacity onto a neutron star is already ruled out. Chen and Colgate [66], however, have recently suggested that the opacity appropriate for a neutron star atmosphere has been

underestimated by several orders of magnitude. Using the opacity of iron at X-ray photon energies, they conclude that the appropriate Eddington limit cannot yet rule out accretion onto a neutron star.

Second, a Crab-like pulsar cannot exist in SN1987A since the emitted magnetic dipole radiation would be too large. The magnetic field and/or the spin rate of a neutron star remnant must be much less than in the case of the Crab and, therefore, much less than is inferred for other young neutron stars. It is possible, however, that although the spin rate of a newly formed neutron star is expected to be high, the timescale for the generation of a significant magnetic field is greater than 10 years. Unfortunately, this timescale is not known with certainty [67].

The experimental measurements [68] of neutrinos from SN1987A indicated the following:

- A total binding energy of $\sim (0.1 - 0.2)M_{\odot}$ was released, indicating, from Fig. 28, a remnant gravitational mass of $(1.14 - 1.55)M_{\odot}$. In addition, about half or more of the binding energy appears to have been released during the first 2 seconds, in agreement with the analysis of the previous section and Fig. 29. The binding energy arguments therefore do not discriminate among the various scenarios we have discussed.
- The average neutrino energy was ~ 10 MeV; to lowest order, this is fixed by the mean free path $\lambda_{\nu}(E_{\nu})$ in the outer regions of the protoneutron star, and also does not shed much light on the internal stellar composition.
- In spite of the fact that most of the binding energy is released during the initial accretion and collapse stage in the first 2 or so seconds after bounce, the neutrino signal continued for a period of at least 12 s. This latter timescale may be significant, since it is also about the time required for the neutrinos initially trapped in the star to leave. However, counting statistics prevented measurement of a longer duration, and this unfortunate coincidence prevents one from distinguishing a model in which negatively-charged hadronic matter appears and a black hole forms from a less exotic model, in which a neutron star still exists. As we have pointed out, the maximum stable mass drops by $\sim 0.2M_{\odot}$ when the trapped neutrinos depart if negatively charged hadrons are present, be they hyperons, kaons or quarks, which could be enough to lead to continued collapse to a black hole.

Observed neutron stars lie in a very small range of gravitational masses. The smallest range that is consistent with all the data [69] runs from $1.34M_{\odot}$ to $1.44M_{\odot}$, the latter value being the accurate measurement of PSR1913+16. Thielemann, et.al. [70] and Bethe and Brown [65] have estimated the gravitational mass of the compact core of SN1987A to be in the range $\sim (1.40 - 1.56)M_{\odot}$, using arguments based on the observed amounts of ejected

^{56}Ni and/or the total explosion energy. This range extends above the largest accurately known value for a neutron star mass, $1.44 M_{\odot}$, so the possibility exists that the neutron star initially produced in SN1987A could be unstable in the cold, deleptonized state. A possible scenario is that such a mass could be stabilized initially when the neutrinos were trapped, but could become unstable when the neutrino concentration dropped to very small values (see Fig. 16, for example). In this case, therefore, SN1987A would have become a black hole once it had deleptonized, and no further signal would be expected.

Note that this scenario for black hole formation in SN1987A is different from that originally proposed by Brown, Bruenn and Wheeler [64], who suggested that long-term accretion on the remnant eventually resulted in the production of a black hole. Although recent 2-dimensional hydrodynamical calculations of supernovae [60, 61] suggest that significant accretion ceases when the supernova shock lifts off, no more than 1 second after bounce, it remains to be seen whether the reverse shocks generated when the shock reaches the low-density hydrogen layers produces significant fallback [71].

6.4 Future detections

A fundamental question is whether or not future neutrino detectors will be able to discriminate among EOS models. The neutrino signal observed in a terrestrial detector is the folding of the emitted neutrino spectrum with the detector characteristics. The latter is a combination of the appropriate microphysical cross sections for neutrino scattering and absorption together with the efficiency function and fiducial (effective) volume of the detector. Another important parameter of the detector is its low-energy threshold, or cutoff. Burrows, Klein and Gandhi [72] list some properties of present, under construction, and future detectors, together with a rough estimate of the numbers of neutrinos that would be observed from a supernova located within our Galaxy (assumed to be at a distance of 10 kpc). Table 17 lists some of the characteristics of the various neutrino telescopes.

In an optimistic scenario, about 10,000 neutrinos will be seen from a typical galactic supernova in a single detector. A crucial question, which has not yet received much attention, is whether the statistical uncertainty in a time-dependent signal can be small enough to adequately differentiate models.

Among the interesting features that could be sought are:

1. Possible cessation of a neutrino signal due to black hole formation.
2. Possible burst or light curve feature associated with the onset of negatively-charged hadrons near the end of deleptonization, whether or not a black hole is formed.

3. Identification of the deleptonization/cooling epochs by changes in luminosity evolution or in neutrino flavor distribution.
4. Determination of a radius-mean-free-path correlation from the luminosity decay time or the onset of neutrino transparency.
5. Determination of the neutron star mass from the universal-binding energy–mass relation.

To realize the above goals, more information about the characteristics of neutrino telescopes must be made widely available. This is especially important in deciphering the time evolution of the neutrino signal (see, for example, Lattimer and Yahil [52]) even if a large number (10,000 or more) of neutrinos are detected in total.

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7 References

References

- [1] A. Burrows and J. M. Lattimer, *Astrophys. J.* **307** (1986) 178.
- [2] A. Burrows, *Ann. Rev. Nucl. Sci.* **40** (1990) 181.
- [3] K. Sato, *Prog. Theor. Phys.* **53** (1975) 595.
- [4] T. J. Mazurek, *Astrophys. Space Sci.* **35** (1975) 117.
- [5] A. Burrows, *Astrophys. J.* **334** (1988) 891.
- [6] V. Thorsson, M. Prakash and J. M. Lattimer, *Nucl. Phys.* **A572** (1994) 693.
- [7] W. Keil and H. T. Janka, *Astronomy and Astrophys.* **296** (1994) 145.
- [8] M. Prakash, J. Cooke and J. M. Lattimer, *Phys. Rev.* **D52** (1995) 661.
- [9] N. K. Glendenning, *Astrophys. J.* **448** (1995) 797.
- [10] J. M. Lattimer, K. A. van Riper, Madappa Prakash and Manju Prakash, *Astrophys. J.* **425** (1994) 802.
- [11] G. E. Brown and H. A. Bethe, *Astrophys. J.* **423** (1994) 659.
- [12] S. E. Thorsett, Z. Arzoumanian, M. M. McKinnon and J. H. Taylor, *Astrophys. J.* **405** (1994) L29.
- [13] J. I. Kapusta, *Finite Temperature Field Theory*, Cambridge University Press, 1985.
- [14] R. C. Tolman, *Proc. Nat. Acad. Sci. USA* **20** (1934) 3; J. R. Oppenheimer and G. M. Volkov, *Phys. Rev.* **55** (1939) 374.
- [15] G. Baym, C. J. Pethick and P. Sutherland, *Astrophys. J.* **170** (1971) 299.
- [16] J. W. Negele and D. Vautherin, *Nucl. Phys.* **A207** (1974) 298.
- [17] M. Prakash, T. L. Ainsworth and J. M. Lattimer, *Phys. Rev. Lett.* **61** (1988) 2518.
- [18] R. B. Wiringa, V. Fiks and A. Fabrocini, *Phys. Rev.* **C38** (1988) 1010.
- [19] M. Prakash, P. J. Ellis, E. K. Heide and S. Rudaz, *Nucl. Phys.* **A575** (1994) 583.

- [20] B. D. Serot and J. D. Walecka, *Advances in Nuclear Physics* **19** ed. J.W. Negele and E. Vogt (Plenum, NY, 1986); B. D. Serot, *Rep. Prog. Phys.* **55** (1992) 1855.
- [21] D. B. Kaplan and A. E. Nelson, *Phys. Lett.* **B175** (1986) 57; **B179** (1986) 409 (E).
- [22] I. Bombaci and U. Lombardo, *Phys. Rev.* **C44** (1991) 1892.
- [23] I. Bombaci, Madappa Prakash, Manju Prakash, P. J. Ellis, J. M. Lattimer and G. E. Brown, *Nucl. Phys.* **A583** (1995) 623.
- [24] C. Gale, G. M. Welke, M. Prakash, S. J. Lee and S. Das Gupta, *Phys. Rev.* **C41** (1990) 1545.
- [25] D. Vautherin and D. M. Brink, *Phys. Rev.* **C5** (1972) 626.
- [26] J. M. Lattimer, C. J. Pethick, M. Prakash and P. A. Haensel, *Phys. Rev. Lett.* **66** (1991) 2518.
- [27] M. Prakash, T. L. Ainsworth, J. P. Blaizot and H. Wolter, in *Windsurfing the Fermi Sea*, Proceedings of the International Conference and Symposium on “Unified Concepts of Many-Body Problems”, Stony Brook, Sept 4-6, 1986, eds. T. T. S. Kuo and J. Speth, (1986) pp 357-381.
- [28] M. Prakash, T. T. S. Kuo, and S. Das Gupta, *Phys. Rev.* **C37** (1988) 2253.
- [29] F. Hoyle and W. A. Fowler, *Astrophys. J.* **132** (1960) 565.
- [30] N.K. Glendenning, *Nucl. Phys.* **A493** (1989) 521.
- [31] J. Ellis, J.I. Kapusta and K.A. Olive, *Nucl. Phys.* **B348** (1991) 345.
- [32] E. K. Heide and S. Rudaz, *Phys. Lett.* **B262** (1991) 375.
- [33] S. A. Chin, *Ann. Phys.* **108** (1977) 301.
- [34] N.K. Glendenning and S.A. Moszkowski, *Phys. Rev. Lett.* **67** (1991) 2414.
- [35] H. D. Politzer and M. B. Wise, *Phys. Lett.* **B273** (1991) 156.
- [36] G. E. Brown, K. Kubodera, M. Rho and V. Thorsson, *Phys. Lett.* **B291** (1992) 355.
- [37] T. Muto and T. Tatsumi, *Phys. Lett.* **B283** (1992) 165; T. Muto, *Prog. Theor. Phys.* **89** (1993) 415; T. Muto *et al.*, *Prog. Theor. Phys. Suppl.* **112** (1993) 221; H. Fujii *et al.*, *Nucl. Phys.* **A571** (1994) 758.

- [38] T. Maruyama, H. Fujii, T. Muto and T. Tatsumi, Phys. Lett. **B337** (1994) 19.
- [39] G. E. Brown, C-H. Lee, M. Rho and V. Thorsson, Nucl. Phys. **A567** (1994) 937.
- [40] R. Büttgen, K. Holinde, A. Müller-Groeling, J. Speth and P. Wyborny, Nucl. Phys. **A506** (1990) 586; A. Müller-Groeling, K. Holinde and J. Speth, Nucl. Phys. **A513** (1990) 557.
- [41] P. J. Ellis, R. Knorren and M. Prakash, Phys. Lett. **B349** (1995) 11.
- [42] R. Knorren, M. Prakash and P.J. Ellis, Phys. Rev. **C52** (1995) 3470.
- [43] J. Schaffner and I.N. Mishustin, preprint (1995) nucl-th/9506011.
- [44] S-J. Dong and K-F. Liu, Nucl. Phys. **B** (Proc. Suppl.) **42** (1995) 322.
- [45] E. Friedman, A. Gal and C.J. Batty, Nucl. Phys. **A579** (1994) 578.
- [46] V.I. Ogievetskij and I.V. Polubarinov, Ann. Phys. (NY) **25** (1963) 358.
- [47] J. Schaffner, A. Gal, I.N. Mishustin, H. Stöcker and W. Greiner, Phys. Lett. **B334** (1994) 268.
- [48] J. Mareš, E. Friedman, A. Gal and B. K. Jennings, Nucl. Phys. **A594** (1995) 311.
- [49] C. J. Horowitz and B. D. Serot, Nucl. Phys. **A368** (1981) 503.
- [50] N. K. Glendenning, Phys. Rev. **D46** (1992) 1274.
- [51] H. Heiselberg, C. J. Pethick and E. F. Staub, Phys. Rev. Lett. **70** (1993) 1355.
- [52] J. M. Lattimer and A. Yahil, Astrophys. J. **340** (1989) 426.
- [53] J. M. Lattimer, M. Prakash, D. Masak and A. Yahil, Astrophys. J. **355** (1990) 241.
- [54] M. Nauenberg and G. Chapline, Astrophys. J. **179** (1973) 277.
- [55] O. V. Maxwell, Astrophys. J. **316** (1987) 691.
- [56] N. Iwamoto, Ann. Phys. **141** (1982) 1.
- [57] S. Chandrasekhar, *An Introduction to the Study of Stellar Structure*, Dover Publications Inc., New York, 1967.
- [58] S. Reddy and M. Prakash, Astrophys. J. (1996), submitted; LANL preprint astro-ph/9512038.

- [59] R. Gandhi and A. Burrows, Phys. Lett. **B246** (1990); **B261** (1991) 519(E).
- [60] A. Burrows and B. Fryxell, Astrophys. J. Lett. **418** (1993) L33; J. Hayes and A. Burrows, Astrophys. J. **450** (1995) 830; A. Burrows and J. Hayes, Phys. Rev. Lett. **76** (1995) 352.
- [61] M. Herant, W. Benz, J. Hix, C. Fryer and S. A. Colgate, Astrophys. J. **435** (1994) 339.
- [62] S. E. Woosley, N. Langer and T. A. Weaver, Astrophys. J. **448** (1995) 315.
- [63] S. E. Woosley and T. A. Weaver, Astrophys. J. Suppl. **101** (1995) 181.
- [64] G.E. Brown, S.W. Bruenn and J.C. Wheeler, Comments Astrophys. **16** (1992) 153.
- [65] H.A. Bethe and G.E. Brown, Astrophys. J. **445** (1995) L129.
- [66] K. Chen and S.A. Colgate, (1995) Los Alamos preprint LA-UR-95-2972.
- [67] A. Muslimov and D. Page, Astrophys. J. Lett. **440** (1995) L77.
- [68] K. Hirata *et al.*, Phys. Rev. Lett. **58** (1987) 1490; R.M. Bionta *et al.*, Phys. Rev. Lett. **58** (1987) 1494.
- [69] S.E. Thorsett, Z. Arzoumanian, M.M. McKinnon and J.H. Taylor, Astrophys. J. **405** (1993) L29; M.H. van Kerkwijk, J. van Paradijs and E.Z. Zuiderwijk, Astron. and Astrophys. **303** (1995) 497; G.E. Brown, J.C. Weingartner and R.A.M.J. Wijers, Astrophys. J. (1995) to be published.
- [70] F.-K. Thielemann, M. Hashimoto and K. Nomoto, Astrophys. J. **349** (1990) 222.
- [71] S. A. Colgate, M. Herant and W. Benz, Phys. Rep. **227** (1993) 157.
- [72] A. Burrows, D. Klein and R. Gandhi, Phys. Rev. **D45** (1992) 3361.

Table 1
Potential model parameters for nuclear matter

EOS	K_0	A	B	B'	σ	C_1	C_2
BPAL1	120	75.94	-30.88	0	0.498	-83.84	23
BPAL2	180	440.94	-213.41	0	0.927	-83.84	23
BPAL3	240	-46.65	39.45	0.3	1.663	-83.84	23
SL1	120	3. 706	-31.155	0	0.453	-41.28	23
SL2	180	159.47	-109.04	0	0.844	-41.28	23
SL3	240	-204.01	72.704	0.3	1.235	-41.28	23

Parameters in Eq. (10) determined by fitting the equilibrium properties of symmetric nuclear matter for some input values of the compression modulus K_0 [17]. All quantities are in MeV, except for the dimensionless σ . The finite-range parameters $\Lambda_1 = 1.5p_F^{(0)}$ and $\Lambda_2 = 3p_F^{(0)}$.

Table 2
Potential model parameters for neutron-rich matter

EOS	K_0	x_0	x_3	Z_1	Z_2
BPAL11		-0.689	0.577	-14.00	16.69
BPAL12	120	-1.361	-0.244	-13.91	16.69
BPAL13		-1.903	-1.056	-1.83	5.09
BPAL21		0.086	0.561	-18.40	46.27
BPAL22	180	-0.410	-0.105	-9.38	24.05
BPAL23		-1.256	-1.358	-11.67	-10.90
BPAL31		0.376	0.246	-12.23	-2.98
BPAL32	240	0.927	-0.227	-11.51	8.38
BPAL33		1.654	-1.112	3.81	13.16
SL12	120	-3.548	-0.5	-13.355	2.789
SL22	180	-0.410	-0.105	9.38	-4.421
SL32	240	-0.442	-0.5	-13.387	2.917

Parameters in Eq. (11); x_0 and x_3 are dimensionless, the remaining quantities are in MeV. For each compression modulus K_0 , the three different choices of the constants yield the potential part of the symmetry energy that varies approximately as \sqrt{u} , u and $2u^2/(1+u)$, respectively, as in the parameterization of Ref. [17]. In all cases, the symmetry energy at the nuclear matter equilibrium density is taken to be 30 MeV. The notations BPAL n_1n_2 and SL n_1n_2 are used used to denote different EOSs; n_1 refers to different values of K_0 , and $n_2 = 1, 2$ and 3 indicate, respectively, a \sqrt{u} , u , and $2u^2/(1+u)$ dependence of the nuclear symmetry potential energy on the density.

Table 3
Pure neutron star properties at finite entropy in the potential models.

EOS	S	$\frac{M_{\max}}{M_{\odot}}$	R (km)	$\frac{n_c}{n_0}$	P_c MeV fm $^{-3}$	T_c MeV	$\lambda \cdot 10^2$	I M_{\odot} km 2
BPAL 22	0	1.896	10.509	7.344	546.6	0.0		87.60
	1	1.941	10.987	6.812	493.0	72.9	2.61	95.73
	2	2.093	12.331	5.392	346.4	138.4		126.60
SL22	0	2.020	10.55	7.05	645.2	0.0		98.97
	1	2.109	11.12	6.37	565.6	117.0	4.31	113.53
	2	2.369	12.59	4.90	429.2	208.2		163.24

R , n_c , P_c , T_c , and I refer to the radius, central density, pressure, temperature, and moment of inertia of the maximum mass star. The coefficient λ (see Eq. (29)) shows the increase in the maximum mass due to thermal effects.

Table 4
 Star properties for matter in beta equilibrium at finite entropy
 in the BPAL potential model.

EOS	S	$\frac{M_{\max}}{M_{\odot}}$	R	$\frac{n_c}{n_0}$	P_c	T_c	$\lambda \cdot 10^2$	I
			km		MeV fm $^{-3}$	MeV		M_{\odot} km 2
BPAL 11	0	1.393	8.219	12.656	843.1	0.0		36.49
	1	1.415	8.540	11.875	786.6	55.9	1.73	39.11
	2	1.489	9.571	9.687	552.4	97.6		48.84
BPAL 12	0	1.454	8.943	10.938	659.5	0.0		43.49
	1	1.475	9.250	10.156	621.5	43.5	1.48	46.35
	2	1.540	10.219	8.594	451.6	82.4		56.38
BPAL 13	0	1.473	9.446	10.156	566.6	0.0		47.16
	1	1.495	9.872	9.375	449.7	39.3	1.60	51.00
	2	1.567	10.840	7.812	376.4	74.1		62.67
BPAL 21	0	1.672	9.172	9.687	766.2	0.0		58.83
	1	1.689	9.437	9.219	712.4	50.5	1.00	61.73
	2	1.739	10.239	7.969	550.9	88.4		71.07
BPAL 22	0	1.722	9.721	8.750	638.3	0.0		66.36
	1	1.735	9.943	8.437	564.2	40.5	0.79	68.78
	2	1.776	10.630	7.500	500.7	77.1		77.64
BPAL 23	0	1.737	10.104	8.281	566.5	0.0		70.13
	1	1.752	10.350	7.969	536.4	36.1	0.88	73.26
	2	1.798	11.120	6.875	409.9	69.5		83.63
BPAL 31	0	1.905	10.107	7.734	652.1	0.0		84.79
	1	1.917	10.280	7.500	629.0	41.3	0.62	87.31
	2	1.952	10.878	6.878	543.5	78.4		95.29
BPAL 32	0	1.933	10.420	7.343	590.2	0.0		90.14
	1	1.943	10.589	7.138	577.7	36.7	0.53	92.52
	2	1.974	11.136	6.506	482.8	71.5		100.17
BPAL 33	0	1.955	10.797	7.000	532.0	0.0		95.35
	1	1.966	11.020	6.719	507.0	33.3	0.49	98.57
	2	1.994	11.518	6.198	454.3	66.0		105.72

Table 5
 Star properties for matter in beta equilibrium at finite entropy
 in the SL potential model.

EOS	S	$\frac{M_{\max}}{M_{\odot}}$	R	$\frac{n_c}{n_0}$	P_c	T_c	$\lambda \cdot 10^2$	I
			km		MeV fm $^{-3}$	MeV		M_{\odot} km 2
SL12	0	1.7423	9.145	9.53	923.3	0.0		62.66
	1	1.7711	9.526	8.91	816.8	71.4	1.86	67.46
	2	1.8710	10.61	7.27	558.2	121.0		85.53
SL22	0	1.890	9.840	8.13	773.5	0.0		80.26
	1	1.920	10.132	7.73	724.9	64.0	1.59	85.15
	2	2.010	11.110	6.56	500.7	113.5		102.60
SL32	0	2.0971	10.572	6.81	689.9	0.0		107.14
	1	2.1211	10.790	6.60	651.6	56.6	1.11	111.60
	2	2.1901	11.549	5.83	532.2	103.2		127.13

Table 6

Star properties for matter with trapped neutrinos ($Y_{Le} = 0.4$) in beta equilibrium at finite entropy in the BPAL potential model.

EOS	S	$\frac{M_{\max}}{M_{\odot}}$	R km	$\frac{n_c}{n_0}$	P_c MeV fm $^{-3}$	T_c MeV	$\lambda \cdot 10^2$	I M_{\odot} km 2
BPAL 11	0	1.376	8.459	11.875	766.3	0.0		36.48
	1	1.396	8.705	11.250	710.6	43.3	1.63	38.78
	2	1.465	9.750	9.219	505.3	78.9		48.30
BPAL 12	0	1.394	8.635	11.406	719.0	0.0		38.37
	1	1.413	8.884	10.781	663.8	41.2	1.59	40.77
	2	1.482	9.919	8.906	478.1	75.7		50.47
BPAL 13	0	1.403	8.782	11.094	681.6	0.0		39.59
	1	1.422	9.035	10.469	620.5	39.7	1.60	42.11
	2	1.492	10.099	8.594	447.3	73.0		52.37
BPAL 21	0	1.641	9.300	9.219	705.4	0.0		57.38
	1	1.656	9.465	8.958	684.7	38.7	0.96	59.40
	2	1.704	10.199	7.917	549.0	72.9		67.80
BPAL 22	0	1.655	9.403	9.062	688.6	0.0		59.03
	1	1.669	9.580	8.750	664.7	37.0	0.93	61.24
	2	1.716	10.330	7.708	519.5	70.3		69.79
BPAL 23	0	1.662	9.505	8.906	665.1	0.0		60.21
	1	1.676	9.677	8.594	636.5	35.9	0.92	62.45
	2	1.723	10.427	7.604	509.2	68.5		71.06
BPAL 31	0	1.855	10.036	7.656	644.6	0.0		79.65
	1	1.867	10.198	7.422	618.4	34.1	0.66	82.01
	2	1.904	10.747	6.812	523.9	66.4		89.78
BPAL 32	0	1.862	10.092	7.578	633.2	0.0		80.74
	1	1.874	10.249	7.344	610.7	33.7	0.64	83.17
	2	1.910	10.820	6.719	514.1	65.3		90.82
BPAL 33	0	1.871	10.203	7.437	609.6	0.0		82.42
	1	1.883	10.330	7.266	604.6	32.8	0.63	84.44
	2	1.918	10.922	6.625	503.7	63.7		92.35

Table 7
MRHA coupling constants and saturation properties

$\frac{\mu_r}{M}$	$\frac{M_{N\text{sat}}^*}{M}$	K_0	C_ω^2	C_σ^2	C_ρ^2
(MeV)					
0.79	0.66	354	180.6	317.5	73.5
1.00	0.73	461	137.7	215.0	81.6
1.25	0.82	264	78.6	178.6	90.8

Table 8
 Star properties for matter in beta equilibrium at finite entropy
 using relativistic models.

$\frac{\mu_r}{M}$	S	$\frac{M_{\max}}{M_{\odot}}$	R	$\frac{n_c}{n_0}$	P_c	T_c	$\lambda \cdot 10^2$	I
			(km)		MeV fm $^{-3}$	MeV		M_{\odot} km 2
0.79	0	2.532	12.62	4.73	432.2	0.0		191.18
	1	2.535	12.75	4.63	416.3	28.9	0.10	193.37
	2	2.542	13.00	4.46	394.9	58.3		197.24
1.00	0	2.305	12.02	5.34	428.2	0.0		151.05
	1	2.311	12.14	5.24	416.6	27.9	0.25	153.19
	2	2.328	12.45	4.98	387.2	56.7		158.82
1.25	0	1.857	10.60	7.29	484.9	0.0		85.63
	1	1.868	10.72	7.19	466.1	29.1	0.56	87.38
	2	1.899	11.19	6.60	419.6	58.8		94.33
GM	0	2.005	10.92	7.14	545.8	0.0		100.6
	1	2.014	11.08	6.95	521.9	31.6	0.47	103.1
	2	2.044	11.56	6.43	458.2	62.6		110.6

The symbol GM refers to the EOS of Ref. [34]. The other three sets are for the MRHA model.

Table 9

Star properties for matter with trapped neutrinos ($Y_{Le} = 0.4$) in beta equilibrium at finite entropy using relativistic models.

$\frac{\mu_r}{M}$	S	$\frac{M_{\max}}{M_{\odot}}$	R	$\frac{n_c}{n_0}$	P_c	T_c	$\lambda \cdot 10^2$	I
			(km)		MeV fm $^{-3}$	MeV		M_{\odot} km 2
0.79	0	2.447	12.28	4.86	460.1	0.0		172.77
	1	2.452	12.29	4.78	448.9	26.4	0.25	174.73
	2	2.471	12.70	4.55	415.7	52.6		181.38
1.00	0	2.222	11.68	5.47	451.2	0.0		135.90
	1	2.228	11.72	5.42	448.5	25.9	0.40	137.27
	2	2.257	12.16	5.09	406.4	51.5		145.39
1.25	0	1.784	10.28	7.50	514.1	0.0		76.29
	1	1.791	10.23	7.42	511.4	27.6	0.75	77.16
	2	1.836	10.89	6.81	448.3	54.6		85.25
GM	0	1.935	10.53	7.41	595.8	0.0		90.13
	1	1.946	10.69	7.22	568.3	30.2	0.58	92.48
	2	1.980	11.19	6.67	496.6	59.0		100.1

Table 10
MRHA coupling constants and saturation properties with hyperons.

$\frac{\mu_r}{M}$	$\frac{M_{N\text{sat}}^*}{M}$	K_0	C_ω^2	C_σ^2	C_ρ^2	x_ω
(MeV)						
0.79	0.76	177	118.7	258.1	84.8	0.660
1.00	0.73	455	133.1	210.3	82.4	0.658
1.25	0.84	228	64.9	174.0	92.6	0.679

In all cases, the ratio of hyperon to nucleon sigma and rho couplings are taken to be equal: $x_\sigma = g_{\sigma H}/g_{\sigma n} = x_\rho = g_{\rho H}/g_{\rho n} = 0.6$.

Table 11
 Star properties for matter, including hyperons, in beta equilibrium
 at finite entropy using relativistic models.

$\frac{\mu_r}{M}$	S	$\frac{M_{\max}}{M_{\odot}}$	R	$\frac{n_c}{n_0}$	P_c	T_c	$\lambda \cdot 10^2$	I
			(km)		MeV fm $^{-3}$	MeV		M_{\odot} km 2
0.79	0	1.638	10.62	7.81	376.7	0.0		67.8
	1	1.639	10.73	7.66	365.6	19.7	0.08	68.6
	2	1.643	10.99	7.39	353.5	42.2		70.0
1.00	0	1.886	12.12	5.64	248.1	0.0		107.4
	1	1.884	12.16	5.64	250.1	16.9	-0.11	106.9
	2	1.878	12.32	5.54	247.9	35.8		106.6
1.25	0	1.407	10.43	8.44	310.3	0.0		51.0
	1	1.414	10.59	8.13	293.1	18.4	0.36	52.5
	2	1.428	10.86	7.81	283.9	39.2		54.5
GM	0	1.544	10.78	7.66	311.4	0.0		63.2
	1	1.551	10.95	7.34	290.5	19.7	0.47	65.2
	2	1.573	11.32	6.88	269.5	41.4		69.2

Table 12

Star properties for matter, including hyperons and trapped neutrinos ($Y_{Le} = 0.4$),
in beta equilibrium at finite entropy using relativistic models.

$\frac{\mu_r}{M}$	S	$\frac{M_{\max}}{M_{\odot}}$	R	$\frac{n_c}{n_0}$	P_c	T_c	$\lambda \cdot 10^2$	I
			(km)		MeV fm $^{-3}$	MeV		M_{\odot} km 2
0.79	0	1.843	11.03	6.60	368.5	0.0		89.5
	1	1.837	11.15	6.46	355.4	17.0	-0.15	89.8
	2	1.831	11.49	6.25	337.7	36.3		91.2
1.00	0	2.066	12.15	5.27	290.2	0.0		126.5
	1	2.063	12.27	5.19	281.5	15.4	-0.11	127.1
	2	2.057	12.56	5.03	266.9	32.2		128.8
1.25	0	1.580	10.48	7.66	355.9	0.0		63.1
	1	1.585	10.61	7.50	344.6	18.0	0.30	64.3
	2	1.599	11.11	6.88	299.7	36.9		64.9
GM	0	1.768	11.11	6.63	334.8	0.0		83.6
	1	1.772	11.21	6.56	332.0	17.5	0.10	84.5
	2	1.776	11.66	6.15	296.7	37.0		88.5

Table 13

Critical density ratio, $u_{crit} = n_{crit}/n_0$, for kaon condensation in the relativistic mean field model, GM, for the neutrino-free and trapped neutrino cases ($Y_{Le} = 0.4$), with and without hyperons.

$a_3 m_s$ (MeV)	model	S	Without hyperons		With hyperons	
			$Y_\nu = 0$	$Y_{Le} = 0.4$	$Y_\nu = 0$	$Y_{Le} = 0.4$
-134	chiral	0	4.15	6.38	9.46	**
	mes. exch.	0	4.54	7.29	**	**
-222	chiral	0	3.15	4.35	4.22	?
	mes. exch.	0	3.59	5.46	**	**
-310	chiral	0	2.49	3.15	2.73	?
	mes. exch.	0	2.86	4.03	3.76	?

The symbol ** indicates that no condensation takes place for this set of couplings.

Table 14

Properties of a star, both without and with trapped neutrinos ($Y_{Le} = 0.4$), which contains neutrons, protons, and kaon condensates in beta equilibrium in the relativistic mean field GM model.

Model	$a_3 m_s$	S	$Y_\nu = 0$				$Y_{Le} = 0.4$			
			$\frac{M_{\max}}{M_\odot}$	R	$\frac{n_c}{n_0}$	I	$\frac{M_{\max}}{M_\odot}$	R	$\frac{n_c}{n_0}$	I
			MeV	(km)	M_\odot km 2		(km)	M_\odot km 2		
chiral	-134	0	1.911	11.39	6.38	98.8	1.934	10.56	7.05	90.6
	-222	0	1.781	9.89	8.62	69.4	1.902	10.60	7.05	88.1
	-310	0	1.779	9.02	9.78	63.4	1.838	9.94	8.05	75.2
mes.	-134	0	1.950	11.36	6.38	102.2	1.935	10.53	7.05	90.3
exch.		1	1.971	11.50	6.25	105.1	1.945	10.74	6.84	92.9
		2	2.015	11.79	5.99	111.3	1.977	11.22	6.37	100.1
mes.	-222	0	1.832	10.65	7.50	81.6	1.928	10.65	6.88	91.4
exch.		1	1.866	11.22	6.72	90.9	1.939	10.78	6.84	92.9
		2	1.946	12.01	5.80	107.8	1.970	11.23	6.38	99.8

Table 15

Ratios of hyperon-meson to nucleon-meson coupling constants, $x_{iH} = g_{iH}/g_{iN}$, where $i = \sigma, \omega$ or ρ , and H is a hyperon species.

Case	$x_{\sigma\Lambda}$	$x_{\omega\Lambda}$	$x_{\rho\Lambda}$	$x_{\sigma\Sigma}$	$x_{\omega\Sigma}$	$x_{\rho\Sigma}$	$x_{\sigma\Xi}$	$x_{\omega\Xi}$	$x_{\rho\Xi}$
1	0.60	0.65	0.60	0.54	0.67	0.67	0.60	0.65	0.60
2	0.60	0.65	0.60	0.77	1.00	0.67	0.60	0.65	0.60
3	0.60	0.65	0.60	0.77	1.00	0.67	0.77	1.00	0.67

Table 16

Maximum masses of stars, M_{\max}/M_{\odot} , with baryonic matter that undergoes a phase transition to quark matter without ($Y_{\nu} = 0$) and with ($Y_{Le} = 0.4$) trapped neutrinos. Results are for a mean field model of baryons and a bag model of quarks. B denotes the bag pressure in the quark EOS.

B (MeV fm $^{-3}$)	Without hyperons		With hyperons	
	$Y_{\nu} = 0$	$Y_{Le} = 0.4$	$Y_{\nu} = 0$	$Y_{Le} = 0.4$
136.6	1.440	1.610	1.434	1.595
150	1.444	1.616	1.436	1.597
200	1.493	1.632	1.471	1.597
250	1.562	1.640	1.506	1.597
No quarks	1.711	1.645	1.516	1.597

Table 17

SUPERNOVA NEUTRINO TELESCOPE CHARACTERISTICS

Detector	Total mass (tonnes) (Fiducial Mass (tonnes))	Composition	Threshold (MeV)	# Events at 10 kpc
CERENKOV:				
KIII	3000 (2140)	H ₂ O	5	370
Super Kamiokande	40,000 (32,000)	H ₂ O	5	5500
SNO	1600/1000	H ₂ O/D ₂ O	5	780
SCINTILLATION:				
LVD	1800 (1200)	Kerosene	5–7	375
MACRO	1000	“CH ₂ ”	10	240
Baksan	330 (200)	“White Spirits” “CH ₂ ”	10	70
LSND	200	“CH ₂ ”	5	70
Borexino	300	(BO) ₃ (OCH ₃) ₃	~ 0.2	200
Caltech	1000	—	2.8	290
DRIFT CHAMBER:				
ICARUS	3600	⁴⁰ Ar	5	120
RADIOCHEMICAL:				
Homestake ³⁷ Cl	610	C ₂ Cl ₄	0.814	4
Homestake ¹²⁷ I	—	NaI	0.664	25
Baksan ³⁷ Cl	3000	C ₂ Cl ₄	0.814	22
EXTRAGALACTIC:				
SNBO	100,000	CaCO ₃	—	10,000
JULIA	40,000	H ₂ O	—	10,000

8 Figure Captions

Fig. 1. The main stages of evolution of a neutron star. Numbers in parentheses refer to the stages discussed in the text.

Fig. 2. Results (left panels for BPAL EOS and right panels for SL interactions) for pure neutron matter. Top panels show the neutron effective mass ratio from Eq. (25) and Eq. (26) versus the density ratio $u = n/n_0$, where $n_0 = 0.16 \text{ fm}^{-3}$ is the equilibrium nuclear density. The center panels show isentropic pressures, and the bottom panels show star masses versus central density ratio at fixed entropy per baryon.

Fig. 3. Results (left panels for BPAL EOS and right panels for SL interactions) for matter in beta equilibrium among n , p , e^- , and μ^- , at an entropy per baryon $S = 1$. Top panels: Individual concentrations $Y_i = n_i/n_b$, where $i = n$, p , e^- and μ^- . Center panels: The electron chemical potential $\mu_e = \mu_\mu = \mu_n - \mu_p$. Bottom panels: Individual contributions to the entropy per baryon.

Fig. 4. Results (left panels for BPAL EOS and right panels for SL interactions) for matter in beta equilibrium among n , p , e^- and μ^- . (See caption to Fig. 2 for further details. Proton effective masses are also shown here.)

Fig. 5. The moment of inertia, I , as a function of density (left panel) and baryonic mass, M_B (right panel). The BPAL22 equation of state is employed for fixed values of the entropy per baryon. The full dots on the curves indicate the maximum gravitational mass.

Fig. 6. Results for the BPAL model with trapped neutrinos at entropy per baryon $S = 1$. The upper panel shows individual concentrations, the center panel gives the leptonic chemical potentials, with $\mu = \mu_e - \mu_{\nu_e}$, and the lower panel separates the nucleon and lepton contributions to the entropy per baryon.

Fig. 7. Stellar temperature, T , as a function of the density ratio u for the MRHA model with $\mu_r/M=1.25$. The full curves are for neutrino free matter, and the dotted curves refer to matter with trapped neutrinos. In the upper (lower) panel, the baryons are nucleons without (with) hyperons.

Fig. 8. Results for matter in beta equilibrium among n , p , e^- , and μ^- in the MRHA model, with $\mu_r/M=1.25$, at an entropy per baryon $S = 1$. (See caption to Fig. 3 for further details.)

Fig. 9. Top panel: Nucleon Landau effective mass ratios versus density ratio for the MRHA

model (with $\mu_r/M=1.25$). Middle panel: Isentropic pressures. Bottom panel: Star mass versus central density ratio at fixed entropy per baryon.

Fig. 10. Results for matter in beta equilibrium among n , p , e^- , μ^- , and trapped neutrinos, in the MRHA model, with $\mu_r/M=1.25$, at an entropy per baryon $S = 1$. Shown are the individual concentrations (top panel), the leptonic chemical potentials (middle panel), and the baryonic and leptonic contributions to the entropy as a function of density.

Fig. 11. Relative fractions and the electron chemical potential for beta-equilibrated matter containing nucleons, hyperons, electrons, and muons in the MRHA model ($\mu_r/M=1.25$) at zero temperature.

Fig. 12. Results for matter in beta equilibrium among nucleons, hyperons, electrons, and muons in the MRHA model with $\mu_r/M=1.25$ at an entropy per baryon $S = 1$. Shown are the individual concentrations (top panel), the electron chemical potential (middle panel), and the baryonic and leptonic contributions to the entropy as a function of density.

Fig. 13. Relative fractions and leptonic chemical potentials for beta-equilibrated matter containing nucleons, hyperons, electrons, muons, and trapped neutrinos in the MRHA model ($\mu_r/M=1.25$) at zero temperature. Here $\mu = \mu_e - \mu_{\nu_e}$.

Fig. 14. Results for beta-equilibrated matter containing nucleons, hyperons, electrons, muons and trapped neutrinos in the MRHA model ($\mu_r/M=1.25$) at an entropy per baryon $S = 1$. Shown are the individual concentrations (top panel), the leptonic chemical potentials (middle panel), and the baryonic and leptonic contributions to the entropy as a function of density.

Fig. 15. Panel (1): Ratio of gravitational mass M_G to baryonic mass M_B as a function of M_B for the GM model. Solid lines are for lepton-rich matter, dashed lines for neutrino-poor matter. A dot at the end of a curve indicates matter with hyperons, a star indicates matter without hyperons. For the neutrino-poor cases, the entropy per baryon is given next to the curves. Panel (2): Gravitational mass as a function of baryonic mass. The symbols are the same as in panel (1).

Fig. 16. Maximum neutron star mass as a function of electron-neutrino fraction Y_{ν_e} in the GM model for matter with and without hyperons, labeled by npH and np, respectively.

Fig. 17. Illustrative plot of the kaon energies ω^\pm in the meson exchange model as a function of the density ratio u . Here the baryons are nucleons. The chemical potential μ

is also shown; the dashed portion of the curve indicates the behavior when kaons are absent.

Fig. 18. Neutrino-free matter in beta equilibrium among nucleons, (thermal) kaons, electrons and muons in the GM model, with the meson-exchange formalism as a function of temperature, T . Results are shown for three different values of the kaon-nucleon sigma term Σ^{KN} . Bottom panel: Critical nucleon density ratio for the onset of kaon condensation. Next to bottom panel: The electron chemical potential $\mu_e = \mu_\mu = \mu_n - \mu_p = \mu_K$. Next to top panel: Thermal kaon to baryon ratio at threshold, for kaon condensation. Top panel: Proton fraction at threshold.

Fig. 19. Results for neutrino-free matter in beta equilibrium among nucleons, kaons, electrons and muons in the GM model for an entropy per baryon $S = 1$. The top panel shows the relative concentrations. The center panel shows the electron chemical potential, and the bottom panel shows the contributions to the total entropy from the strongly interacting particles and the leptons, respectively.

Fig. 20. Results for neutrino-trapped matter in beta equilibrium among nucleons, (thermal) kaons, electrons, and muons in the GM model as a function of temperature, T , with three different values of the kaon-nucleon sigma term Σ^{KN} . Bottom panel: Critical nucleon density ratio for the onset of kaon condensation. Next to bottom panel: The electron chemical potential $\mu_e = \mu_\mu = \mu_n - \mu_p = \mu_K$. Next to top panel: Thermal kaon to baryon ratio. Top panel: Proton fraction at threshold, for kaon condensation.

Fig. 21. Results for neutrino-trapped matter in beta equilibrium among nucleons, kaons, electrons, and muons in the GM model for an entropy per baryon $S = 1$. The top panel shows the relative concentrations. The center panel gives the leptonic chemical potentials, with $\mu = \mu_e - \mu_{\nu_e}$, and the bottom panel shows the contributions to the total entropy from the strongly interacting particles and the leptons, respectively.

Fig. 22. Results for zero-temperature matter in beta equilibrium among nucleons, hyperons, kaons, electrons, and muons. The chiral model with $a_3 m_s = -222$ MeV is used in conjunction with the mean field GM description of the baryons. (a) Relative fractions $Y_i = n_i / (\sum_B n_B)$. (b) Baryon Dirac effective masses, the kaon chemical potential $\mu = \mu_e$, and the scalar field σ . (c) Baryon scalar densities. (d) Condensate amplitude, θ , in degrees.

Fig. 23. Particle fractions for model HS81 in conjunction with the kaon meson-exchange formalism for different choices of the Σ and Ξ coupling constants. Panels (1), (2), and (3) correspond to parameter sets 1, 2, and 3 of Table 15, respectively.

Fig. 24. Gravitational mass vs. baryonic mass for matter with and without kaons in the lepton-rich ($Y_{Le} = 0.4$) and neutrino-poor stages ($Y_\nu = 0$). The chiral model with $a_3 m_s = -222$ MeV is used in conjunction with the mean field GM description for the nucleons. The range in neutron star masses that is metastable during deleptonization is indicated.

Fig. 25. Maximum neutron star mass as a function of electron-neutrino fraction Y_{ν_e} for matter with and without kaons, labeled by np and npK, respectively. (See caption to Fig. 21 for further details.)

Fig. 26. Individual concentrations for matter in beta equilibrium among nucleons, hyperons, quarks, electrons, and muons, employing the mean field GM model in the baryon sector and a bag model for the quarks. The top panel shows the neutrino-free case and the bottom panel the results with trapped neutrinos. The quark phase cavity pressure $B = 200$ MeV fm $^{-3}$.

Fig. 27. Quark-hadron phase transition boundaries in beta-equilibrated matter as a function of the bag pressure, B . In the top panel, the hadrons are nucleons and, in the lower panel, nucleons and hyperons. The onset of a quark-hadron mixed phase occurs at a density ratio u_1 , and a pure quark phase begins at u_2 . Also shown is the central density ratio, u_c , of the maximum mass star.

Fig. 28. Binding energy versus baryon mass for nucleons-only matter (np), matter with nucleons and hyperons (npH), and matter with nucleons and kaons (npK). The stars, dots, and triangles mark the maximum mass configurations. The lower envelope is for an EOS that is causal above a transition density of $n_t = 0.3$ fm $^{-3}$ and for the GM EOS below n_t .

Fig. 29. Enclosed gravitational mass versus baryon mass. Two generic compositional cases are displayed: nucleons-only matter and matter with hyperons. Solid curves are for neutrino-rich matter with $Y_{Le} = 0.4$ at an entropy per baryon $S = 2$. Dashed curves refer to cold catalyzed neutrino-free matter.

Fig. 30. Electron concentrations as a function of density for neutrino-free matter with various assumptions for the stellar composition, as indicated. The quark phase cavity pressure $B = 200$ MeV fm $^{-3}$. Arrows indicate central densities of $1.44 M_\odot$ stars. Differences of each curve from $Y_{Le} = 0.4$ show the net Y_{ν_e} lost at each density during cooling.



























































